GAs for aerodynamic shape design II: multiobjective optimization and multi-criteria design

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Abstract

The lecture focuses on multi-objective genetic algorithms with hybrid capabilities, and on their application to multi-criteria design problems. A short introduction to multi-point aerodynamic shape design is given, and the advantages of a multi-objective optimization approach to this problem are outlined. The introduction of basic concepts of multi-objective optimization is followed by the description of a multiple objective genetic algorithm. Some techniques for efficiency improvement are introduced; in particular, the gradient based technique for hybrid optimization is extended to multi-objective design problems. Application examples are reported related both to single and multi-element airfoil design in high-lift conditions, and to transonic wing design.

1 Introduction

The aerodynamic design problem can be defined as the determination of the shape of bodies that satisfy design goals and constraints that can be either of aerodynamic or geometrical and structural nature [1].

Design problems are often characterized by several conflicting requirements that have to be satisfied at the same time.

For example, the main goal of the aerodynamic design of a wing for a transport aircraft is to minimize the drag in cruise condition, but this is not enough to obtain an efficient transport wing. Several additional criteria must be met such as buffet boundary high enough to permit cruising at design lift coefficients, no pitch-up tendencies near stall, no unsatisfactory off-design performances [2]. Furthermore, several non-aerodynamic requirements have to be taken into account, such as weight and structural constraints.

On the other hand, the main goal of a high lift system could appear simpler at a first glance, as, in this case, the objective of the aerodynamic design is to achieve maximum lift without massive flow separation [3]. As a matter of fact, the problem is even more complicated, because a high-lift system has a strong impact on wing weight, complexity and costs and its integration into a wing requires a true multidisciplinary approach.

When conflicting requirements have to be satisfied at the same time, the usual approach is to reduce the multiobjective problem into a classical single objective one. Here, instead, will be described a different technique based on direct solution of the multiobjective problem. Basic concepts and definitions of multiobjective optimization are given in the following, and an approach to its resolution based on genetic algorithms is developed.

2 Multiple objective optimization

In mathematical terms, a Multi-objective Optimization Problem (MOP) may be defined as:

$$\min_{\mathbf{x}\in X} \boldsymbol{f}(\mathbf{x}) \tag{1}$$

where $\boldsymbol{f} \triangleq (f_1(\mathbf{x}), \ldots, f_n(\mathbf{x}))$ is a vector of m real valued objective functions, $\mathbf{x}$ is a vector of n decision variables and

$$X \triangleq \{\mathbf{x} \mid \mathbf{x} \in \mathbf{R}^n,\ \ g_i(\mathbf{x}) \le 0,\ i = 1, \ldots, p,\ \ h_j(\mathbf{x}) = 0,\ j = 1, \ldots, q \ \text{ and } \ \mathbf{x} \in S\}$$

is the feasible solution set, $g_i(\mathbf{x})$ and $h_j(\mathbf{x})$ are real valued functions representing the constraints and S is a subset of $\mathbf{R}^n$ representing any other form of constraint (for example a discrete set). The ideal solution of such a problem is a point where each objective function assumes its best possible value. This ideal solution in most cases does not exist due to the conflicting nature of the objectives. Hence, solutions to these problems have to be a compromise between the various requirements. An usual approach to find a compromise solution is the reduction of the multiobjective problem into a classical single objective one through a weighted combination of the objective functions. The drawback of this technique is that the solution obtained depends on the arbitrary choice of the relative weights assigned to the objectives. It would be interesting, instead, to find all the best compromise solutions available, and choose 'a posteriori' the solution best fitted to the problem. This approach naturally leads to a new definition of optimality: a feasible solution to a multiobjective optimization problem is said Pareto optimal, or non dominated, if, starting from that point in the design space, the value of any of the objective functions cannot be improved without deteriorating at least one of the others.

All feasible solutions to a multi-objective problem can thus be classified into dominated and non dominated (Pareto optimal) solutions, and the set of globally non dominated solutions of a multi-objective problem is called Pareto front. These are all possible alternative solutions to the problem, which meet the requirements at different level of compromise. In this way, the arbitrary choice regarding the weights to attribute to each different design criteria is avoided. Consequently, the first step in the solution of a multi-objective problem consists in finding (or approximating) this set or a representative subset. Afterwards the decision maker's preference may be applied to choose the best compromise solution from the generated set.

The natural ordering of vector valued quantities is basic for Pareto optimality. To define the notion of domination let $\mathbf{f} = (f_1, \ldots, f_m)$ and $\mathbf{g} = (g_1, \ldots, g_m)$ be two real-valued vectors of m elements; $\mathbf{f}$ is partially less than $\mathbf{g}$ (in symbols $\mathbf{f} <_{\mathsf{p}} \mathbf{g}$) if:

$$\forall i \in 1, \ldots, m, \ \ f_i \le g_i, \ \text{ and } \ \exists i : f_i < g_i \tag{2}$$

If $\mathbf{f} <_{\mathsf{p}} \mathbf{g}$, we say that $\mathbf{f}$ dominates $\mathbf{g}$. Consequently, a feasible solution $\mathbf{x}^*$ is said a Pareto optimal solution of the problem in equation 1 if and only if it does not exist another $\mathbf{x} \in X$ such that $\boldsymbol{f}(\mathbf{x}) <_{\mathsf{p}} \boldsymbol{f}(\mathbf{x}^*)$.