

Contents

1	Multi-dimensional upwind residual distribution method	1
1.1	Introduction	1
1.2	Residual distribution schemes for a scalar conservation law	2
1.2.1	Scalar advection	2
1.2.2	Numerical approximation of the solution	3
1.2.3	Residual distribution	5
1.2.4	Linear advection problem	6
1.2.5	Non-linear advection problem	8
1.2.6	Design principles	9
1.2.7	Scalar advection schemes	10
1.3	Residual distribution schemes for hyperbolic systems of conservation laws .	12
1.3.1	System schemes	12
1.3.2	Multi-dimensional high resolution schemes	14
1.4	Residual distribution applied to the Euler equations	15
1.4.1	Euler equations	15
1.4.2	Conservative linearization	16
1.4.3	Invariance for similarity transformations	17
1.4.4	Preconditioning and hyperbolic-elliptic splitting	18
1.5	Residual distribution applied to the Navier-Stokes equations	20
1.5.1	Navier-Stokes equations	20
1.5.2	Galerkin discretization of the viscous terms	20
1.5.3	Turbulence modelling	22
2	Residual distribution schemes and the entropy condition	24
2.1	Introduction	24
2.2	Residual distribution: one-dimensional case	24
2.2.1	Scalar advection scheme	25
2.2.2	Sonic point and trapped shock waves	25

2.2.3	One dimensional entropy fix	26
2.2.4	Hyperbolic systems of conservation laws	29
2.2.5	Entropy fix for one-dimensional hyperbolic system	30
2.3	Shock capturing analysis - the multi-dimensional case	32
2.3.1	Scalar advection: Burgers equation	32
2.3.2	Hyperbolic system: Euler equations	32
2.4	Multi-dimensional entropy fix	34
2.5	Numerical tests	35
2.5.1	Mach 3 wind tunnel with step	36
2.5.2	Mach 3 wind tunnel with triangular bump	37
2.6	Conclusion	38
3	A shock fix for residual distribution schemes	39
3.1	The carbuncle phenomenon	39
3.2	Carbuncle phenomenon with residual distribution method	41
3.3	Cures against the carbuncle phenomenon	43
3.4	Residual distribution shock fix	44
3.5	Numerical results	45
3.5.1	Inviscid flow over a circular cylinder	45
3.5.2	Inviscid flow over the ARD vehicle	45
4	Verification and validation of the accuracy of the residual distribution method for steady Euler and Navier-Stokes simulation	48
4.1	Introduction	48
4.2	Inviscid subsonic flow over sine bump: $\mathbf{M}_{in} = \mathbf{0.4}$	49
4.2.1	Measures of numerical error	49
4.2.2	Results with RD solver	50
4.2.3	Comparison with FV results	52
4.2.4	Conclusion	53
4.3	Inviscid flow over NACA 0012 airfoil	54
4.3.1	Grids and boundary conditions	54
4.3.2	Subsonic flow over NACA 0012 airfoil : $\mathbf{M}_{\infty} = \mathbf{0.5}$, $\alpha = \mathbf{3}^{\circ}$	54
4.3.3	Transonic flow over NACA 0012 airfoil : $\mathbf{M}_{\infty} = \mathbf{0.85}$, $\alpha = \mathbf{1}^{\circ}$	64
4.3.4	Conclusion	69
4.4	Inviscid flow over ONERA M6 wing	70
4.4.1	Grids and boundary conditions	70
4.4.2	Inviscid flow over M6 wing : $\mathbf{M}_{\infty} = \mathbf{0.3}$, $\alpha = \mathbf{0}^{\circ}$	71

4.4.3	Inviscid flow over M6 wing : $M_\infty = 0.84$, $\alpha = 3.06^\circ$	74
4.4.4	Conclusion	75
4.5	Turbulent transonic flow over RAE 2822 airfoil	78
4.5.1	Conclusion	83
	Conclusions	84
	Acknowledgements	85
	Bibliography	86