Aerodynamic Shape Optimization Using the Adjoint Method

Antony Jameson

Department of Aeronautics & Astronautics
Stanford University
Stanford, California 94305 USA

Lectures at the Von Karman Institute, Brussels
Febuary 6, 2003

Abstract

These Lecture Notes review the formulation and application of optimization techniques based on control theory for aerodynamic shape design in both inviscid and viscous compressible flow. The theory is applied to a system defined by the partial differential equations of the flow, with the boundary shape acting as the control. The Frechet derivative of the cost function is determined via the solution of an adjoint partial differential equation, and the boundary shape is then modified in a direction of descent. This process is repeated until an optimum solution is approached. Each design cycle requires the numerical solution of both the flow and the adjoint equations, leading to a computational cost roughly equal to the cost of two flow solutions. Representative results are presented for viscous optimization of transonic wing-body combinations.

1 Introduction: Aerodynamic Design

The definition of the aerodynamic shapes of modern aircraft relies heavily on computational simulation to enable the rapid evaluation of many alternative designs. Wind tunnel testing is then used to confirm the performance of designs that have been identified by simulation as promising to meet the performance goals. In the case of wing design and propulsion system integration, several complete cycles of computational analysis followed by testing of a preferred design may be used in the evolution of the final configuration. Wind tunnel testing also plays a crucial role in the development of the detailed loads needed to complete the structural design, and in gathering data throughout the flight envelope for the design and verification of the stability and control system. The use of computational simulation to scan many alternative designs has proved extremely valuable in practice, but it still suffers the limitation that it does not guarantee the identification of the best possible design. Generally one has to accept the best so far by a given cutoff date in the program schedule. To ensure the realization of the true best design, the ultimate goal of computational simulation methods should not just be the analysis of prescribed shapes, but the automatic determination of the true optimum shape for the intended application.

This is the underlying motivation for the combination of computational fluid dynamics with numerical optimization methods. Some of the earliest studies of such an approach were made by Hicks and Henne [1,2]. The principal obstacle was the large computational cost of determining the sensitivity of the cost function to variations of the design parameters by repeated calculation of the flow. Another way to approach the problem is to formulate aerodynamic shape design within the framework of the mathematical theory for the control of systems governed by partial differential equations [3]. In this view the wing is regarded as a device to produce lift by controlling the flow, and its design is regarded as a problem in the optimal control of the flow equations by changing the shape of the boundary. If the boundary shape is regarded as arbitrary within some requirements of smoothness, then the full generality of shapes cannot be defined with a finite number of parameters, and one must use the concept of the Frechet derivative of the cost with respect to a function. Clearly such a derivative cannot be determined directly by separate variation of each design parameter, because there are now an infinite number of these.

Using techniques of control theory, however, the gradient can be determined indirectly by solving an adjoint equation which has coefficients determined by the solution of the flow equations. This directly corresponds to the gradient technique for trajectory optimization pioneered by Bryson [4]. The cost of solving the adjoint equation is comparable to the cost of solving the flow equations, with the consequence that the gradient with respect to an arbitrarily large number of parameters can be calculated with roughly the same computational cost as two flow solutions. Once the gradient has been calculated, a descent method can be used to determine a shape change which will make an improvement in the design. The gradient can then be recalculated, and the whole process can be repeated until the design converges to an optimum solution, usually within 10 - 50 cycles. The fast calculation of the gradients makes optimization computationally feasible even for designs in three-dimensional viscous flow. There is a possibility that the descent method could converge to a local minimum rather than the global optimum solution. In practice this has not proved a difficulty, provided care is taken in the choice of a cost function which properly reflects the design requirements. Conceptually, with this approach the problem is viewed as infinitely dimensional, with the control being the shape of the bounding surface. Eventually the equations must be discretized for a numerical implementation of the method. For this purpose the flow and adjoint equations may either be separately discretized from their representations as differential equations, or, alternatively, the flow equations may be discretized first, and the discrete adjoint equations then derived directly from the discrete flow equations.

The effectiveness of optimization as a tool for aerodynamic design also depends crucially on the proper choice of cost functions and constraints. One popular approach is to define a target pressure distribution, and then solve the inverse problem of finding the shape that will produce that pressure distribution. Since such a shape does not necessarily exist, direct inverse methods may be ill-posed. The problem of designing a two-dimensional profile to attain a desired pressure distribution was studied by Lighthill, who solved it for the case of incompressible flow with a conformal mapping of the profile to a unit circle [5]. The speed over the profile is

$$q = \frac{1}{h} \left| \boldsymbol{\nabla} \phi \right|,$$

where ϕ is the potential which is known for incompressible flow and h is the modulus of the mapping function. The surface value of h can be obtained by setting $q = q_d$, where q_d is the desired speed, and since the mapping function is analytic, it is uniquely determined by the value of h on the boundary. A solution exists for a given speed q_∞ at infinity only if

$$\frac{1}{2\pi} \oint q \, d\theta = q_\infty,$$

and there are additional constraints on q if the profile is required to be closed.

The difficulty that the target pressure may be unattainable may be circumvented by treating the inverse problem as a special case of the optimization problem, with a cost function which measures the error in the solution of the inverse problem. For example, if p_d is the desired surface pressure, one may take the cost function to be an integral over the the body surface of the square of the pressure error,

$$I = \frac{1}{2} \int_{\mathcal{B}} (p - p_d)^2 d\mathcal{B},$$

or possibly a more general Sobolev norm of the pressure error. This has the advantage of converting a possibly ill posed problem into a well posed one. It has the disadvantage that it incurs the computational costs associated with optimization procedures.

The inverse problem still leaves the definition of an appropriate pressure architecture to the designer. One may prefer to directly improve suitable performance parameters, for example, to minimize the drag at a given lift and Mach number. In this case it is important to introduce appropriate constraints. For example, if the span is not fixed the vortex drag can be made arbitrarily small by sufficiently increasing the span. In practice, a useful approach is to fix the planform, and optimize the wing sections subject to constraints on minimum thickness.

Studies of the use of control theory for optimum shape design of systems governed by elliptic equations were initiated by Pironneau [6]. The control theory approach to optimal aerodynamic design was first applied to transonic flow by Jameson [7–12]. He formulated the method for inviscid compressible flows with shock waves governed by both the potential flow and the Euler equations [8]. Numerical results showing the method to be extremely effective for the design of airfoils in transonic potential flow were presented in [13,14], and for three-dimensional wing design using the Euler equations in [15]. More recently the method has been employed for the shape design of complex aircraft configurations [16,17], using a grid perturbation approach to accommodate the geometry modifications. The method has been used to support the aerodynamic design