

## Modelling multiphase flow with an Eulerian approach

Simon Lo, CD-adapco, UK  
simon.lo@uk.cd-adapco.com

### 1 Eulerian multiphase model

#### 1.1 Definition

Multiphase flows refer to flows of several fluids in the domain of interest. In general, we associate fluid phases with gases, liquids or solids and as such some simple examples of multiphase flows are: air bubbles rising in a glass of water, sand particles carried by wind, rain drops in air. In fact, the definition of ‘phase’ can be generalised and applied to other fluid characteristics such as size and shape of particles, density, temperature, etc. With this broader definition, multiple phases can be used to represent the entire size distribution of particles in several size groups or ‘phases’ of a multiphase model. Mass transfer between the particle phases can therefore be used to represent particles moving between different size groups due to break-up and coalescence processes.

In the Eulerian multiphase model, the phases are treated as interpenetrating continua coexisting in the flow domain. Equations for conservation of mass, momentum and energy are solved for each phase. The share of the flow domain occupied by each phase is given by its volume fraction and each phase has its own velocity, temperature and physical properties. Interactions between phases due to differences in velocity and temperature are taken into account via the inter-phase transfer terms in the transport equations. In the solution method described here, all the phases share a common pressure field.

The Eulerian multiphase model provides a general framework for all types of multiphase flows; both dispersed (e.g. bubble, droplet, and particle flows) and stratified (e.g. free-surface flows) flows can be modelled. In this lecture we will focus on modelling of dispersed flows.

#### 1.2 Main equations

The main equations solved are the conservation of mass, momentum and energy for each phase.

##### Continuity

The conservation of mass for phase  $k$  is:

$$\frac{\partial}{\partial t}(\alpha_k \rho_k) + \nabla \cdot (\alpha_k \rho_k u_k) = \sum_{i=1}^N (\dot{m}_{ki} - \dot{m}_{ik}) \quad (1.1)$$

where  $\alpha_k$  is the volume fraction of phase  $k$ ,  $\rho_k$  is the phase density,  $u_k$  is the phase velocity,  $\dot{m}_{ki}$  and  $\dot{m}_{ik}$  are mass transfer rates to and from the phase, and  $N$  is the total number of phases. The sum of the volume fractions is clearly equal to unity.

$$\sum_k \alpha_k = 1 \quad (1.2)$$

## Momentum

The conservation of momentum for phase  $k$  is:

$$\frac{\partial}{\partial t}(\alpha_k \rho_k u_k) + \nabla \cdot (\alpha_k \rho_k u_k u_k) - \nabla \cdot (\alpha_k (\tau_k + \tau_k^t)) = -\alpha_k \nabla p + \alpha_k \rho_k g + M \quad (1.3)$$

where  $\tau_k$  and  $\tau_k^t$  is the laminar and turbulence shear stresses,  $p$  is pressure and  $M$  is the sum of the interfacial forces. We will examine the interfacial forces in Section 2 below.

## Energy

The conservation of energy for phase  $k$  is:

$$\frac{\partial}{\partial t}(\alpha_k \rho_k e_k) + \nabla \cdot (\alpha_k \rho_k u_k e_k) - \nabla \cdot (\alpha_k \lambda_k \nabla T_k) = Q \quad (1.4)$$

where  $e_k$  is the phase enthalpy,  $\lambda_k$  is the thermal conductivity and  $Q$  is the interfacial heat transfer.

## 2 Interfacial forces

The interfacial forces between the phases are: drag, lift, virtual mass, turbulent drag, wall lubrication and momentum transfer associated with mass transfer, hence

$$M = F_D + F_L + F_M + F_T + F_W + \sum_{i=1}^N (\dot{m}_{ki} u_i - \dot{m}_{ik} u_k) \quad (2.1)$$

### 2.1 Drag force

The drag force on a single particle,  $D$ , can be calculated from: