STRUCTURAL MODELING AND DYNAMICS

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1 Introduction

Structural modelling and dynamics deals with the behaviour of deformable bodies subjected to the action of loading. It constitutes a traditional topic in engineering therefore it is expected that all engineers are familiar with the relevant theory techniques. So the only reason for having such a chapter in this lectures series is to provide a background in support of the next lectures which focus of the aeroelastic behaviour of wind turbines.

We start with the presentation of basic concepts of structural mechanics such as stresses and strains and their relation in describing the response of deformable bodies. Since there are more than one ways to describe the deformed state of a solid body, we will introduce the principal of virtual work as the tool for relating strains and stresses in a compatible way. Next we discuss the formulation of the equilibrium equations as defined by Newton's law and their relation to stresses. The choice of appropriate boundary and initial conditions is then indicated within the context of variational considerations, in particular the Galerkin approach.

Section 3 gives the basics of structural dynamics and includes the derivation of the dynamic equilibrium equations, the eigenvalue analysis of dynamic systems and their simulation in the time domain.

Section 4 summarizes the basic results of the classical beam theory. Beam approximations play an important role in engineering applications and certainly for wind turbine analysis is what we almost exclusively use. We derive the equilibrium equations in differential form for a combined loading state including tension, bending and torsion.

Section 5 gives a short presentation of the Finite Element Method as applied to beam like structures. We start by defining the interpolation procedure. Then we formulate the dynamic equations in variational form and finally we derive the discrete dynamic equations in the form of a dynamic system.

Finally the present lecture closes with data reduction where the theory of thin walled sections is used in order to determine the sectional structural properties of a beam like structure.

2 Basic Background on Structural Mechanics

In this section we are going to review the basics of structural mechanics primarily in view of the work presented latter on. References for further reading could include the books by Green & Zerna [1] and Timoshenko & Goodier [2]

2.1 Strains and Stresses

Structural mechanics deals with solid bodies that deform under specific loading. Deformations are related to displacements and strains while loads are related to tractions and stresses. The relation between loading and deformation can be formulated in different ways depending on the type of analysis one wishes to follow. Of course at the end the results should be compatible so the aim of this section is to clarify the details of the different approaches.

Consider a solid body $\mathcal{B}$ described with respect to a fixed reference system $[O;x,y,z]_G$ [1] . Deformation of $\mathcal{B}$ will result in the displacement of all of its points. Let $\vec{u}$ denote the displacement field:

$$\vec{u}=\vec{x}-\vec{x}_o \tag{2-1}$$

where $\vec{x}$ and $\vec{x}_o$ denote the positions of the point in the deformed and undeformed state respectively. In order to have a reference to the point considered, the initial undeformed position $\vec{x}_o$, also denoted in some cases $\vec{\xi}$, is used.

Given a displacement field $\vec{u}$ the next step is formulate a description for the deformation or strain. Consider a line element of length $\ell_o = dx$ subjected to uni-directional tension. If the two end points are displayed by u and u+du respectively then the so-called engineering strain is defined as the change in length $\ell-\ell_o=(\ell_o+du)-\ell_o=du$ divided by the original length ℓ^o :

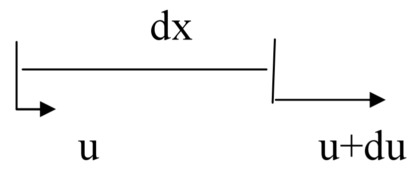

Figure 2-1: Definition of engineering strain

$$\varepsilon_E = \frac{\ell-\ell_o}{\ell_o} = \frac{(u+du)-u}{dx} = \frac{du}{dx} = u' \tag{2-2}$$

where u' denotes the derivative of u in the direction of deformation. The line element we have taken would result from a slender structure with a constant cross section S which is small compared to the length of the structure. Assuming a linear material, the deformation will be caused by loading acting in the direction of deformation. By taking the load per unit surface being constant over the cross section, we define the stress σ_E : $\sigma_E = E\varepsilon_E$ .with E being Young's modulus. If the load causing the deformation is denoted as Q then by applying the principle of virtual work it follows that: $Q=\sigma_E S=ESu'$. For a virtual displacement δu the work done by Q should be equal to the internal virtual work done by the stress:

$$Q\delta u=\sigma_E S.\delta\varepsilon_E \ell_0 = \sigma_E S\frac{\delta u}{\ell_0}\ell_0 \Rightarrow Q=\sigma_E S \tag{2-3}$$

The above derivation is suitable when strains are small. If this is not the case instead of (2-2) we introduce strain in an incremental form:

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[1] Subscript "G" will be used to denote the so called global or fixed system system.