STABILITY ANALYSIS AND CONTROL

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1 Introduction

Aeroelastic stability is of particular importance in the design of modern wind turbines. It affects both reliability and safety. During its early stages of development wind energy industry was not taking this aspect into account basically relying on designing wind turbines in a conservative manner. However experience indicated clearly that stability is a serious issue. In particular it was realized that when upsizing wind turbines to 500kW, after a short period of operation cracks started to develop on the blade skin [1]. The fact that most wind turbines at that time were stall regulated contributed a lot. As theory has proven, stall regulated wind turbines suffer from negative aerodynamic damping and therefore if the structural damping gets low, the risk of entering into instability is high. Such events gave to stability very high priority. A lot of theoretical work was done since then covering most of the original lack of knowledge. In the present lecture we will review this aspect of aeroelasticity analysis of wind turbines.

We will start by introducing some basic concepts of linear stability analysis. Floquet's theory for periodic systems will be presented as the general background for this kind of analysis. Next the rotational transformation is detailed which is a powerful tool in analyzing rotor systems. Then before discussing some indicative results, in section 3, we are going to briefly present some topics related to more advanced models. The main reason for presenting them here is because of the results that follow. In this context we consider: the 2nd order theory of beams and the Timoshenko beam model as regards structural modeling, while on the aerodynamic modeling we describe free wake vortex methods. Results on stability are presented in section 4 while in section 5 the relevance of control in view of enhancing stability is discussed again through typical results.

2 Basic concepts in Linear Stability Analysis

Stability analysis deals with the characterization of the response of a dynamic system to an external excitation. From linear theory we know that the response of a dynamic system to an impulsive excitation will trigger its eigenmodes. If the system can damp all eigenmodal responses then the system is stable. If the amplitude of one or more eigenmodal responses is amplified then the system is unstable. The limit situation in which the amplitudes remain constant is referred to as neutral. The ability of a system to damp external excitations as well as the level of damping plays a critical role in reliability and safety. As the level of damping decreases, the amplitudes of the loading increase and therefore the lifetime decreases. In the particular case of wind turbines, material aging as well as the dependence of material damping on temperature for the blades calls for increased attention.

2.1 Floquet's theory

The dynamic equations can be reformulated into a 1^{st} order set of equations by introducing collectively the displacements and their velocities as degrees of freedom. For a typical dynamic system:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{Q} \tag{2-1}$$

if $\mathbf{p}^T = \begin{bmatrix} \dot{\mathbf{q}}^T & \mathbf{q}^T \end{bmatrix}$

$$\begin{bmatrix} \mathbf{M} & 0 \\ 0 & \mathbf{I} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\mathbf{q}} \\ \dot{\mathbf{q}} \end{bmatrix} + \begin{bmatrix} \mathbf{C} & \mathbf{K} \\ -I & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\mathbf{q}} \\ \mathbf{q} \end{bmatrix} = \cdot \begin{bmatrix} \mathbf{Q} \\ 0 \end{bmatrix} \text{ or } \tilde{\mathbf{M}}\dot{\mathbf{p}} + \tilde{\mathbf{K}}\mathbf{p} = \tilde{\mathbf{Q}} \tag{2-2}$$

Because $\tilde{\mathbf{M}}$ will be always invertible, by setting $\mathbf{A} = -\tilde{\mathbf{M}}^{-1}\tilde{\mathbf{K}}$ and $\mathbf{B}=\tilde{\mathbf{M}}^{-1}\tilde{\mathbf{Q}}$ it follows that

$$\dot{\mathbf{p}} = \mathbf{A}\mathbf{p} + \mathbf{B} \tag{2-3}$$

If A, B do not depend on time, then the eigenvalues of A provide all the necessary information as discussed in Lecture 1. In rotor applications this is not the case. The coefficients depend on time. In the special case that the system is periodic linear stability analysis can be still performed based on Floquet's theory [2].

Assume that $\mathbf{A}(t-T) = \mathbf{A}(t)$ then the solution of the homogeneous problem $\dot{\mathbf{p}} = \mathbf{A}(t)\mathbf{p}$ should be of the form: $\mathbf{p}(t) = \Re(t, t_0)\mathbf{p}(t_0)$ where $\Re(t, t_0)$ denotes the state transition matrix. By substituting the above representation into the dynamic equations, we obtain:

$$\frac{d}{dt}\Re(t, t_0) = \mathbf{A}(t)\Re(t, t_0), \quad \Re(t_0, t_0) = \mathbf{I} \tag{2-4}$$

The state transition matrix gives the required solution for (2-3):

$$\mathbf{p}(t) = \Re(t, t_0)\mathbf{p}(t_0) + \int_{t_0}^{t} \Re(t, \tau)\mathbf{B}(\tau)d\tau \tag{2-5}$$

Taking into account periodicity,

$$\frac{d}{dt}\Re(t+T, t_0) = \mathbf{A}(t+T)\Re(t+T, t_0) = \mathbf{A}(t)\Re(t+T, t_0) \tag{2-6}$$

So in order $\Re(t+T, t_0), \Re(t, t_0)$ to be solutions of the same equation: $d\Re / dt = \mathbf{A} \cdot \Re$ they must be linearly related:

$$\Re(t+T, t_0) = \mathbf{C} \cdot \Re(t, t_0) \tag{2-7}$$

$\mathbf{C}$ is a constant matrix containing all the information we need. If (2-4) is integrated over one period it follows that $\mathbf{C} = \Re(t_0 + T, t_0)$. This represents a quite computationally intensive process as it involves matrix integration. For a system described by N d.o.f., for each d.o.f the equations must be integrated over one period. So depending on the size of the system this task can become exceedingly expensive.

Having determined $\mathbf{C}$, eigenvalue analysis can deduce its modal matrix $\mathbf{\Phi}$ and its eigenvalue matrix $\mathbf{\Lambda}_C$: $\mathbf{C} = \mathbf{\Phi}\mathbf{\Lambda}_C\mathbf{\Phi}^{-1}$. Getting back to (2-6), let: