Introductory Concepts in Mesh Generation

N. P. Weatherill, O. Hassan and K. Morgan
Department of Civil Engineering, University of Wales Swansea, UK

1. Introduction

The computational grid on which a discretisation of an equation is based is a fundamental component of the numerical solution procedure. The general requirements for a grid are:

1. The computational grid must discretise the domain Ω and all boundaries Γ. This closely links the grid to the geometry of the domain. The distribution of points in the domain must, firstly, adequately represent the geometry of the region.

2. The nodes and elements of the grid should be so inter-related, or connected, that appropriate approximations to mathematical operators i.e. the discrete approximation, can be computed in an efficient manner. This determines the inherent structure within the grid.

3. The spatial distribution of nodes should provide well-shaped elements, with no large discontinuities in nodal and element spacing. The accuracy of a numerical discretisation is dependent upon the properties of the underlying spatial discretisation. For example, the classical second order accurate finite difference representation of a second spatial derivative is only second order if the grid point spacing is uniform. Furthermore, it can be shown that other spatial derivatives only achieve second order accuracy if the grid lines are approximately orthogonal. Hence, the spatial discretisation of the domain should be achieved without discontinuous grid point spacing and without the introduction of highly skewed cells or elements.

4. The distribution of nodes in the grid should allow for an appropriate resolution of features in the solution. For accurate simulations, it is necessary that there are adequate points in regions where there is high activity, i.e. where the unknown variables change rapidly.

5. Ideally, the minimum number of nodes should be used to achieve a given level of accuracy of the solution of the governing equations.

These requirements on a grid make the generation of suitable computational grids a non-trivial problem[1].

A classification of grid types arises from how the nodes of a mesh are connected to form the grid elements. A ***structured grid*** consists of a set of nodes which naturally map into an array. Neighbouring nodes in the grid are neighbouring elements in the grid node matrix. As such, the connections between nodes to form elements are related in a structured manner to the neighbour elements in a matrix.

Definition A grid is called structured if its connectivity is of a finite difference type, i.e.(i,j) ordering, with (i+1,j) a direct neighbour. This naturally extends to 3 dimensions.

For an ***unstructured mesh***, the nodes cannot be represented in such a manner and additional information has to be provided. For any particular node, the connection with other nodes to form the grid elements must be defined explicitly in a connectivity matrix. Hence, in addition to the storage of the co-ordinates of the grid nodes it is necessary to store the node numbers which when connected form the grid elements.

Definition A mesh is called unstructured if its connectivity is not of a structured format

This chapter will concentrate of some of the basic methods and issues related to structured grid generation.