

Delaunay triangulation. Theoretical aspects, element quality and robustness

P.L. George

INRIA, Domaine de Voluceau, BP 105, Le Chesnay Cedex, France

Abstract

Delaunay triangulation algorithms play an important role in Delaunay based mesh generation methods. Various theoretical issues about this triangulation can be used as themselves or after some adjustments for mesh generation purposes. Element quality and robustness issues are of utmost importance in this respect.

Note : The following materials include materials extracted from [10], Chapters 2 and 3 and from [7], Chapter 2 and 7 together with original discussions.

Introduction

Automatic unstructured mesh generation is a field which received attention since the early 70th. Initially, mesh generation procedures were developed mostly in two dimensions (J.A. George for an advancing-front method). Then, quadtree and octree (in three dimensions) based methods were investigated (M.S. Shephard at the R.P.I.) together with Delaunay type methods (mostly in two dimensions in terms of computing but in higher dimensions for theoretical issues (and a few tentatives of computer procedures, even in three dimensions)). About Delaunay methods, it could be observed that the initial interest for this class of methods was due to its capability of triangulating a set of points (thus solving a convex hull problem). It is then not surprising to find the first references mostly among the Computational Geometry people (which are not explicitly mentioned here) rather than among the engineering or aerospace people (say, T. Baker, N.P. Weatherill, INRIA's people and many others).

In this course, we focus in detail in a Delaunay based method. To this end, issues about triangulation problems are recalled before going to mesh generation problems (at the same time the main differences between these two problems are made precise). Theoretical issues together with numerical issues are discussed in various parts of the course.

1 Delaunay triangulation

The literature about Delaunay triangulation is rather abundant. For its application for meshing problem, for instance in CFD, one can see the course of N.P. Weatherill, [18], in this series. In the following, we like to gives some additional material about the theoretical aspects of such a triangulation with the objective of finding information directly usable for the actual computer implementation.