Summary

The recent achievements in the investigation of mechanisms of aerodynamic sound generation by vortex ring is presented. The theory of vortex ring noise based on the careful description of vortex ring eigen-oscillations in Euler approximation is presented. The results obtained in these directions permitted understanding the mechanism of sound generation by 3-D vortex and formulating a new concept of sound radiation by vortex structures. It appeared that the noise of even one vortex which seems at first sight to be a rather simple oscillating system was in fact a highly complex random process with quite definite peculiarities which can be recorded experimentally and predicted theoretically. In this lecture we consider the theoretical part of the problem.

1. Introduction

The existing problem of aircraft noise restriction is connected first of all with its adverse effect on people. This problem became urgent at the end of the 20th century due to the sharp increase of aircraft fleet and the simultaneous increase of plant power [1]. Resources of further noise reduction within the limits of traditional approaches are connected with serious technical problems in realizing different methods of its reduction and this places the acoustic characteristics of current aircrafts in one row with the most important criteria of their competitive ability. Therefore the further work in this direction requires proposing new approaches and concepts based on a more profound understanding the physical processes responsible for noise generation by turbulent flows.

Since appearance of jet passenger airplanes in the fifties the main noise source has become the engine exhaust jet. For reliable and accurate prediction of the sound field in this case not only the knowledge of mean fluctuation characteristics of the turbulent flow is required which can be often measured and predicted, but also the knowledge of space and time correlation scales, connection between the disturbances of different scales, turbulent structure, etc. [2,3]. Thus, the main problems in describing the sound generation by turbulence are connected first of all with our restricted possibilities in understanding the turbulence itself. Therefore for understanding the process of noise generation by turbulent flows it is necessary to have, as a reference, at least one quite understandable situation in which the processes of forming disturbances in the vortex and their connection with the sound field could be exactly revealed experimentally and well predicted theoretically "from the first principles" where possible.

As an example of such a "simple" flow it was proposed to consider a free vortex ring [4]. Vortex ring is a well known and very popular object of fluid dynamics. Its investigations have begun in the last century when a vortex ring has been considered as a model of the vortex theory of atoms being under development [5-6]. Though the quantum theory have made insignificant many ideas being developed in that period, the vortex ring seems to remain one of the most interesting and convenient objects for investigations in hydrodynamics [7]. Really, this vortex is open to experimental research and at the same time its behaviour can be described within the limits of principal equations of the continuous medium. The most important of it is that being generated, this vortex is developing only under the effect of its own dynamics and is not affected by rigid boundaries. This permits its using for investigations of many problems of hydroaerodynamics in a pure form. Despite the fact that the vortex ring is a rather popular model object in aerodynamics its application in aeroacoustics has begun comparatively recently. Kambe was the first who began the aeroacoustics experiments with vortex rings [8]. However, in his works the principal attention was paid to noise radiation produced by *macrovariations* of the vortex flow: collision of two rings [8,9], ring flight close to edge or cylinder [10-12]. We consider another problem concerns aeroacoustics of an individual vortex ring in a free space, where the sound radiation is connected with *microvariations* of the vortex flow (vortex core eigen-oscillations). In this approach separate vortex structure itself is an independent source of sound.

Even in such a simple case when sound radiation is made by an individual isolated turbulent vortex ring the problem remains to be extremely complex. Since the real vortex ring is a complex turbulent formation [7,13] (Fig.1), the question is first of all the following: what is the source of acoustic radiation in such a system: turbulence in the ring envelope (ring envelope or "atmosphere" is the so called ellipsoidal part of stream moving together with the core), vorticity shedding in a wake or separate modes of the vortex core and if so, what modes exactly. It appear that the core eigen-oscillations are the acoustic radiation cause and examination of eigen-modes permits not only explaining the radiation mechanism but also understanding the cause of "atmosphere" turbulization [12].

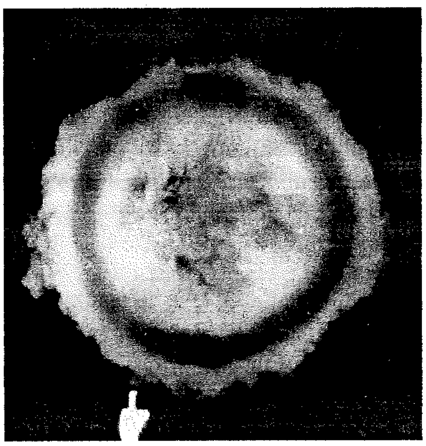

Fig.1 Photo of turbulent vortex ring (front view)

Lets consider what tasks were to be solved for a complete description of the processes guiding the aerodynamic sound generation in a one single vortex only. First of all the exhausting relations connecting the unsteady three-dimensional vorticity dynamics with the sound field are to be obtained. This task is connected, on the one hand, with a careful description of all the possible unsteady disturbances in the vortex ring and, on the other hand, with calculating the sound fields generated by vortical fluctuations and with establishing those oscillations which are the most effective in the sound radiation.

As it is known, the unsteady movement of vortices in a compressible fluid is accompanied by a quadrupole sound radiation [13]. If the characteristic flow Mach number is small and vorticity is localized in some region with a characteristic size much less than the sound wave length, then the sound source is expressed through the unsteady velocity field which is calculated to the approximation of an incompressible fluid. Thus, vortex ring sound field predictions in a *weakly compressible* fluid is based on careful descriptions of vortex eigen-oscillation in an ideal *incompressible* fluid. The oscillations of vortex ring have a close analogy - Kelvin's oscillations of Rankin's cylindrical vortex (vortex column). The similarity of the mean flow of thin vortex ring with almost circular cross-section, small in comparison with the vortex radius, and a cylindrical vortex with circular cross section and infinite curvature radius seems to be the main cause why the vortex ring oscillations (for computation this task is much more complex) have not been considered in a complete form until recent time (see the review of this problem in section 3). It appeared however that many vortex ring modes were different from the respective oscillations of the cylindrical vortex already in the main approximation. The eigen-oscillation structure change appeared not only an unexpected curious fact. This difference involved a number of important consequences: from a multitude instability to peculiarities of acoustic radiation by vortices. Thus, the numerous number of eigen oscillations which are silent for cylindrical vortex appears to be effective sound radiators for vortex ring. These modes are localized close to one and the same frequency and it leads to narrow-band stochastic feature of total sound radiation.

The next fundamental question concerns the energetics of the noise generation process and in particular the mechanism of excitation of sound-generating eigen-oscillations in the vortex ring. The oscillation energy in a separate vortex cannot appear from the outside, since the only energy reservoir is the mean flow itself. Consequently the possibility of unsteady fluctuation appearance must be found and just of those scales which could be responsible for radiation. Therefore revealing the instability mechanisms is the fundamental moment for closing the dynamic part of the problem and hence for understanding the noise generation process being realized. Thus, the theory of vortex ring noise presented here based on three coupled problems – (i) vortex ring eigen-oscillations in incompressible fluid, (i) sound fields accompanying the core oscillations in weekly compressible fluid and (iii) mechanisms of vortex ring instability, produced the energy transfer from steady flow to sound-generating disturbances. These questions we consider in present lecture.

The only criterion separating the real radiation mechanism from a number of possible ones is the experiment. Therefore the main research direction is connected with an experimental diagnostics of high-frequency fluctuations of the vortex core and with measurements of the sound field generated by vortex. The experimental diagnostics of the vortex ring core oscillations (visualization), measurements of the mean flow parameters and the acoustic experiment allow comparing the experimental and theoretical data, dropping some possible radiation mechanisms and finding that sound radiation mechanism which seems to be actually realized in a separate turbulent vortex. These problems are consider in the second lecture [14].

2. Governing equations

Description of disturbances evolution in the cores of localized steady vortices is a separate rather complex problem. For its solution a new approach [15,16] was worked out which is based on using the displacement field ε as the main function. Traditionally for describing disturbances in vortical flows the velocity field or vorticity field is used. Convenience of introducing the new variable - the displacement field ε is connected with the fact that it directly describes each vortex filament deformation. Thus, small displacement of fluid particles from points $\boldsymbol{r}$ to points $\boldsymbol{r}+\varepsilon(\boldsymbol{r})$, $\nabla\varepsilon=0$, keeping to the conditions of freezing vortical filament into displacement field ε, gives vorticity disturbance $\Omega=\nabla\times(\varepsilon\times\Omega_0)$ in a linear approximation.

The system of equations for an ideal incompressible fluid written in the language of displacement field is transformed at $\boldsymbol{r}\in\mathrm{M}$ into the form convenient for solving tasks with localized vortices [17]:

$$\frac{\partial}{\partial t}\nabla\times\varepsilon+\nabla\times[\nabla\times(\varepsilon\times V_0)]-\nabla\times(\varepsilon\times\Omega_0)=0,$$

$$\nabla\varepsilon=0\ , \tag{2.1}$$

$$\mathrm{v}=\nabla\times\frac{1}{4\pi}\int\frac{\nabla'\times(\varepsilon\times\Omega_0)}{|\boldsymbol{r}-\boldsymbol{r}'|}d\boldsymbol{r}' \tag{2.2}$$

where M is the region in which vorticity is concentrated, $G(M)$ is its boundary at which the condition

$$(\partial\varepsilon/\partial t+\nabla\times(\varepsilon\times V_0)-\mathrm{v})\cdot\boldsymbol{n}=0,\quad \boldsymbol{r}\in G(\mathrm{M}) \tag{2.3}$$

is fulfilled.

This system combines the advantages of Helmholtz equation for vorticity which allows searching