

Proper Orthogonal Decomposition: an overview

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1 Introduction

Collecting very large amounts of data by numerical simulations or experimental approaches is a common situation in almost any scientific field. There is therefore a great need to have specific post-processing techniques able to extract from these large quantities of high dimensional data, synthetic information essential to understand and eventually to model the processes under study. The Proper Orthogonal Decomposition (POD) is one of the most powerful method of data-analysis for multivariate and non linear phenomena. Essentially, POD is a linear procedure that takes a given collection of input data and creates an orthogonal basis constituted by functions estimated as the solutions of an integral eigenvalue problem known as a Fredholm equation (see equation 18). These eigenfunctions are by definition (equation 16) characteristic of the most probable realizations of the input data. Moreover, it can be shown that they are optimal in terms of the representation of energy present within the data (see § 4.3).