FREE TO TUMBLE TECHNIQUE: PRACTICAL ASPECTS

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1. Post processing of experimental data

The experimental analysis of wing rock dynamics for a slender delta wing will be considered (free-to-roll experiments) as an example for post processing of experimental data.

The model is supposed to be free to rotate about the longitudinal body axis. The roll angular displacement ϕ is measured by a low friction transducer mounted on the rotating shaft. Highly swept delta wing planforms may exhibit an oscillatory mode (wing rock) in which the aircraft model performs a self-excited large amplitude oscillation in roll.

The dynamic equilibrium can be modeled as:

$$I_X\ddot{\phi} + a_0\phi + a_1\dot{\phi} + a_2\left|\dot{\phi}\right|\dot{\phi} + a_3\phi^3 + a_4\phi^2\dot{\phi} = 0$$

that can be rewritten in adimensional form

$$\phi'' + \hat{a}_0\phi + \hat{a}_1\phi' + \hat{a}_2\left|\phi'\right|\phi' + \hat{a}_3\phi^3 + \hat{a}_4\phi^2\phi' = 0$$

where

$$\left(^{\prime}\right) = d/d\hat{t} \quad \hat{t} = t/t^* \quad t^* = b/2V$$

and

$$\hat{a}_0 = \frac{a_0}{I_X}\cdot t^{*2} \quad \hat{a}_1 = \frac{a_1}{I_X}\cdot t^* \quad \hat{a}_2 = \frac{a_2}{I_X} \quad \hat{a}_3 = \frac{a_3}{I_X}\cdot t^{*2} \quad \hat{a}_4 = \frac{a_4}{I_X}\cdot t^*$$

The time derivatives of ϕ can be obtained with numerical differentiation of the time history $\phi(t)$:

$$\dot{\phi}_i = \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta t} + O\left(\Delta t^2\right) \quad \ddot{\phi}_i = \frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{\Delta t^2} + O\left(\Delta t^2\right)$$

and the rolling moment coefficient is derived as $C_l(t) = \frac{I_X}{qSb} \cdot \ddot{\phi}(t)$.

The coefficients $\hat{a}_0, \hat{a}_1, ..., \hat{a}_4$ can be identified by means of least squares approximation of experimental data with coherent initial conditions (N samples over the sampling period T):

$$\begin{cases} f_a = a_0\phi + a_1\dot{\phi} + a_2\left|\dot{\phi}\right|\dot{\phi} + a_3\phi^3 + a_4\phi^2\dot{\phi} \\ f_e = -I_X\ddot{\phi} \end{cases}$$

$$s^2 = \sum_{i=1}^{N} s_i^2 = \sum_{i=1}^{N}\left(f_{a_i} - f_{e_i}\right)^2$$

The residual s^2 must be minimized and coefficient of determination r^2 must be bounded:

$$\frac{d\sum_{i=1}^{N} s_i^2}{da_0} = 0 \quad \frac{d\sum_{i=1}^{N} s_i^2}{da_1} = 0 \quad \ldots \quad \frac{d\sum_{i=1}^{N} s_i^2}{da_4} = 0$$

$$r_{\min}^2 < r^2 = \frac{\sum_{i=1}^{N}\left(\phi_i^{est} - \bar{\phi}\right)^2}{\sum_{i=1}^{N}\left(\phi_i^{\exp} - \bar{\phi}\right)^2} < r_{\max}^2$$

Hence, the unknown coefficients for the differential model equation can be obtained from the solution of a classical matricial formulation:

$$\left[A\right] \cdot \left\{a_0, a_1, a_2, a_3, a_4\right\}^T = \left\{B\right\}$$

where the matrix [A] and the vector {B} are defined by the minimization of the residuals over the sampling period i.e. for a complete experimental data set including oscillatory build-up and oscillatory steady state.