COMPUTATIONAL METHODS TO DETERMINE AERODYNAMIC STABILITY DERIVATIVES

Paul Weinacht, James DeSpirito, Jubaraj Sahu and Sidra Silton
U.S. Army Research Laboratory
Aberdeen Proving Ground, MD 21005, USA

1. Introduction

The assessment of the free-flight performance and stability of symmetric missiles requires the determination of the complete set of static and dynamic aerodynamic coefficients. Much of the prior work has focused on the prediction of a subset of the complete aerodynamics[1-7], often just the static aerodynamics. However, methods for prediction of all the necessary dynamic derivatives for projectile applications are becoming increasingly accessible.[8,9] Currently, there appears to be two types of computational fluid dynamics (CFD) approaches for addressing the complete aeroprediction problem. One approach involves coupling of the aerodynamics and rigid body dynamics to simultaneously predict the aerodynamics and free-flight motion.[10] This approach has been dubbed the "virtual fly-out" or "free-response" method since it mimics aspects of the free-flight motion. Like its experimental counterpart, the primary purpose of the virtual fly-out technique is the determination of the aerodynamics, although the method also produces the free-flight motion as a by-product of the computation. An alternative approach decouples the rigid body dynamics from the aerodynamics. The practice of decoupling the aerodynamics from the rigid body dynamics and assuming that the aerodynamics of a body undergoing an arbitrary motion can be obtained through a linear sum of the aerodynamics due to a subset of elemental motions is a fundamental part of existing aeroprediction, testing and trajectory prediction methodologies. Here, the elemental motions necessary to excite the aerodynamics of interest are produced by specific prescribed or "forced" motions. Because of the existing analog with testing methods, this approach is referred to as the "virtual wind tunnel" approach as a means of distinguishing this methodology from the "virtual fly-out" approach. The primary advantage of the virtual wind tunnel approach is efficiency. Physical fidelity of the results is of utmost importance and, in this regard, the virtual wind tunnel approach may offer additional benefits compared with the virtual fly-out approach, since important aerodynamic effects are easily isolated and quantified. However, if the aerodynamics are highly nonlinear and coupled to the motion, the "virtual fly-out" approach may be required.

The current paper focuses on the prediction for aerodynamic derivatives for symmetric missiles. The force and moment expansion for this sub-class of flight vehicles is less complex than that of more complicated flight vehicles. The force and moment expansion for these vehicles is described in Section 2. The virtual wind tunnel or "forced motion" approach is then described in Section 3. Both steady and unsteady (time-dependent) methods are described. Section 4 presents the virtual fly-out or "free-response" approach. Summary remarks are then made in Section 5.

2. Force and Moment Expansion for Symmetric Missiles

It is common in many aerodynamics applications to use a body-fixed, non-rolling coordinate system to describe both the dynamics and the system of forces and moments that act on the flight vehicle.[11] The non-rolling coordinate system allows the description of the vehicle dynamics to be simplified for certain classes of flight vehicles that possess particular types of geometric symmetry. Rotationally symmetric flight vehicles (vehicles that are symmetric when rotated axially through 120 degrees or less) which are the focus of the current research, represent one class of vehicles where the non-rolling frame has been effectively (and traditionally) used. For more complicated geometries, such as aircraft, the advantages of the non-rolling frame are reduced and other coordinate frames such as a completely body-fixed coordinate system are typically used.

In the current effort, the primary reason for initially describing the aerodynamic forces and moments using the non-rolling coordinate system is the fact that the description is well established for symmetric flight vehicles. The non-rolling coordinate frame is an orthogonal right-handed system $(\tilde{x}, \tilde{y}, \tilde{z})$ centered at the body center of gravity (CG). The $\tilde{x}$-axis is aligned along the projectile longitudinal axis with the positive direction oriented toward the projectile nose. The $\tilde{z}$-axis is "initially" oriented downward with the $\tilde{x} - \tilde{z}$ plane perpendicular to the ground. The angular motion of the non-rolling coordinate frame is such that, with respect to an inertial frame, the $\tilde{x}$-component of the coordinate frame's angular velocity is zero. Although the time-dependent orientation of the non-rolling frame may be difficult to visualize, the non-rolling frame is essentially equivalent to the "fixed-plane" coordinate system for small amplitude motions. In the fixed-plane coordinate system, the $\tilde{x} - \tilde{z}$ plane remains perpendicular to the ground for all time. The total angular velocity of the flight vehicle can be described in terms of its angular velocity components $(p, \tilde{q}, \tilde{r})$ along the $\tilde{x}$, $\tilde{y}$ and $\tilde{z}$ axes, respectively. The angular velocity of the non-rolling frame can be described in terms of the transverse angular velocities $\tilde{q}$ and $\tilde{r}$ because the angular velocity of the non-rolling frame along the $\tilde{x}$-axis is always zero. The flight body may, however, have a non-zero spin rate, p, about its longitudinal axis. Further details about these coordinate frames are discussed in Ref. 11.

2.1 The Transverse Moment Expansion

The moment expansion for a rotationally symmetric missile in the non-rolling coordinate frame is shown in Eq. 1. This moment expansion is similar to the moment proposed by Murphy.[11] The moment formulation uses complex variables to separate the moment components, $\tilde{C}_m$ and $\tilde{C}_n$, that are oriented along the $\tilde{y}$ and $\tilde{z}$ axes, respectively. The third moment component, the roll moment, can be handled separately and is discussed in a subsequent subsection.

$$\tilde{C}_m + i\tilde{C}_n = \left[\left(\frac{p\ell}{V}\right)C_{n_{p\alpha}} - iC_{m_\alpha}\right]\tilde{\xi} + C_{m_q}\tilde{\mu} - iC_{m_{\dot{\alpha}}}\tilde{\xi}' \tag{1}$$

In the moment expansion, the pitching moment coefficient slope, C_{m_α}, and the coefficient $C_{m_{\dot{\alpha}}}$ represent moments that are proportional to the complex yaw, $\tilde{\xi}$, and yawing rate, $\tilde{\xi}'$, respectively.

The complex yaw and yawing rate are defined below. (In the analysis presented here, there is no need to distinguish between pitch and yaw and the terms may be interchanged. The usage follows that of Murphy.[11])

$$\tilde{\xi} = \frac{\tilde{v} + i\tilde{w}}{V} \tag{2}$$