VKI Feb 06

# Optimal shape design

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**Abstract**

These pages content more details than can possibly be covered during the lectures. On the other hand the second lecture will cover materials not given here pertaining to computer implementations and methods

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# 1 Introduction

In mathematical terms, an optimal shape design requires the optimization to one or several criteria $\{E_i(x)\}_1^I$ which depend on design parameters $x \in X$ which define the shape of the system within the admissible set of values $X$.

Multi-criteria. Optimization is a difficult field in itself of which we shall retain only the min-max idea,

$$\min_{x\in X}\{J(x)\ :\ \ E_i(x) \le J(x),\ \ i=1,...,I\}$$

and the Pareto minimization

$$\min_{x\in X}\sum_{i=1}^{I}\alpha_i E_i(x).$$

For some suitable values of $\alpha_i \in (0,1)$, both problems are equivalent and solve in some intuitive sense the multi-criteria problem. Standard optimization and control theory is applied to these derived problems.

Optimal control for distributed systems is a branch of optimization for problems which involve a parameter or control variable $u$, a state variable $y$ and a partial differential equation $A$ (with boundary conditions $b$), to define $y$ in a domain $\Omega$:

$$\min_u\{J(u,y)\ :\ A(x,y,u)=0\ \ \forall x\in\Omega,\ \ b(x,y,u)=0\ \ \forall x\in\partial\Omega\}.$$

For example,

$$\min_u\{\int_\Omega (y-1)^2\ :\ \ -\Delta y=0\ \ \forall x\in\Omega,\ \ y|_{\partial\Omega}=u\} \qquad (1)$$