

Optimal shape design

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Abstract

These pages content more details than can possibly be covered during the lectures. On the other hand the second lecture will cover materials not given here pertaining to computer implementations and methods

Contents

1	Introduction	2
2	Examples	4
2.1	Minimum weight of structures	4
2.2	Wing drag optimization	5
2.3	Synthetic jets and riblets	11
2.4	Stealth wings	11
2.5	Optimization of a stealth wing	14
2.6	Optimal breakwater	14
2.7	Two academic test cases: nozzle optimization	17
3	Existence of solutions	17
3.1	Topological optimization	17
3.2	Sufficient conditions for existence	18
4	Solution by optimization methods	19
4.1	Gradient methods	20
4.2	Newton methods	21
4.3	Constraints	22
4.4	A constrained optimization algorithm	23
5	Sensitivity analysis	24
5.1	Sensitivity analysis for the nozzle problem	25
6	Discretization with triangular elements	27
6.1	Sensitivity of the discrete problem	29

7	Numerical issues	31
7.1	Implementation problems	31
7.2	Independence from the cost function	31
7.3	Addition of geometrical constraints	32
7.4	Automatic differentiation	33
8	Appendix: Optimal design for Navier-Stokes flows	34
8.1	Optimal shape design for Stokes flows	34
8.2	Optimal shape design for Navier-Stokes flows	35

1 Introduction

In mathematical terms, an optimal shape design requires the optimization to one or several criteria $\{E_i(x)\}_1^I$ which depend on design parameters $x \in X$ which define the shape of the system within the admissible set of values X .

Multi-criteria. Optimization is a difficult field in itself of which we shall retain only the min-max idea,

$$\min_{x \in X} \{J(x) : E_i(x) \leq J(x), \quad i = 1, \dots, I\}$$

and the Pareto minimization

$$\min_{x \in X} \sum_{i=1}^I \alpha_i E_i(x).$$

For some suitable values of $\alpha_i \in (0, 1)$, both problems are equivalent and solve in some intuitive sense the multi-criteria problem. Standard optimization and control theory is applied to these derived problems.

Optimal control for distributed systems is a branch of optimization for problems which involve a parameter or control variable u , a state variable y and a partial differential equation A (with boundary conditions b), to define y in a domain Ω :

$$\min_u \{J(u, y) : A(x, y, u) = 0 \quad \forall x \in \Omega, \quad b(x, y, u) = 0 \quad \forall x \in \partial\Omega\}.$$

For example,

$$\min_u \left\{ \int_{\Omega} (y - 1)^2 : -\Delta y = 0 \quad \forall x \in \Omega, \quad y|_{\partial\Omega} = u \right\} \quad (1)$$