Lecture 2. STABILITY FOR FLOW CONTROL

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2 LINEAR STABILITY ANALYSIS

In this lecture, the stability of steady, laminar, incompressible flows is considered as background to the lecture on Laminar Flow Control. The essential ideas are little changed by compressibility. The notes are self-contained and complete. As such, they provide a complete reference base and much more material than can be presented in one lecture.

The notes concentrate on *bounded shear flows* that are characterized by plane Poiseuille flow and boundary-layer flow. Inviscid free shear layers and wall jets are briefly discussed in Section 2.9. The experimental results of Nishioka et al. (1975, 1980, 1981) in plane Poiseuille flow, which compare with the classic results of Klebanoff et al. (1959, 1962) in the boundary layer, show that the basic instability and transition mechanisms in plane Poiseuille flow and the Blasius boundary layer are identical.

The analysis is initiated by formulating the stability problem for a general basic state and then simplifying the problem for one-dimensional (1-D) basic states with linear three-dimensional (3-D) disturbances. The role of 2-D and 3-D disturbances is described in Section 2.6. Energy methods are described in Section 2.7. Inviscid mechanisms are discussed in Sections 2.8 and 2.8. The rest of the Chapter is devoted to viscous mechanisms.

2.1 The General Basic State with Disturbances

Dimensionless quantities are introduced by using a suitable reference length, L, reference velocity, U_∞, reference time, L/U_∞, and reference pressure, ρU_∞^2. In terms of these normalizing variables, the incompressible Navier-Stokes equations with constant properties become:

$$\nabla \cdot \vec{v} = 0 \tag{2.1}$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\nabla p + \frac{1}{R} \nabla^2 \vec{v} \tag{2.2}$$

where the Reynolds number, R, is given by

$$R = \frac{U_\infty L}{\nu} \tag{2.3}$$

The normalizing length, L, will be defined for specific cases as the analysis proceeds.

Since we are considering the class of stationary basic states, we assume that each flow quantity $\vec{v}(x,y,z,t)$ and $p(x,y,z,t)$ is the sum of a basic-state term, $\vec{V}(x,y,z)$ and $P(x,y,z)$, and a fluctuating term $\vec{v}'(x,y,z,t)$ and $p'(x,y,z,t)$, that is

$$\vec{v}(x,y,z,t) = \vec{V}(x,y,z) + \vec{v}'(x,y,z,t), \quad p(x,y,z,t) = P(x,y,z) + p'(x,y,z,t) \tag{2.4}$$

Expressing each flow quantity in the form of Eq. (2.4) and substituting these expressions into Eqs. (2.1) and (2.2), gives

$$\nabla \cdot \vec{V} + \nabla \cdot \vec{v}' = 0 \tag{2.5}$$

$$\vec{V} \cdot \nabla \vec{V} + \nabla P - \frac{1}{R}\nabla^2 \vec{V} + \frac{\partial \vec{v}'}{\partial t} + \vec{v}' \cdot \nabla \vec{V} + \vec{V} \cdot \nabla \vec{v}' + \vec{v}' \cdot \nabla \vec{v}' + \nabla p' - \frac{1}{R}\nabla^2 \vec{v}' = 0 \tag{2.6}$$

The basic-state quantities are always solutions of the Navier-Stokes equations by themselves so Eqs. (2.5) and (2.6) can be separated into basic-state and disturbance-state equations.

Basic State

$$\nabla \cdot \vec{V} = 0 \tag{2.7}$$

$$\vec{V} \cdot \nabla \vec{V} + \nabla P - \frac{1}{R}\nabla^2 \vec{V} = 0 \tag{2.8}$$

Disturbance State

$$\nabla \cdot \vec{v}' + 0 \tag{2.9}$$

$$\frac{\partial \vec{v}'}{\partial t} + \vec{v}' \cdot \nabla \vec{V} + \vec{V} \cdot \nabla \vec{v}' + \vec{v}' \cdot \nabla \vec{v}' + \nabla p' - \frac{1}{R}\nabla^2 \vec{v}' = 0 \tag{2.10}$$

The basic state is described by Eqs. (2.7) and (2.8) and reduces to whatever specific form is required to analyze the basic flow. The unsteady, nonlinear equations, Eqs. (2.9) and (2.10), illustrate that the disturbance state need not satisfy the Navier-Stokes equations. These equations are time-dependent and one could define a set of mean-flow equations that contain these terms along with Eqs. (2.7) and (2.8). However, for arbitrary