THE SCALE-ADAPTIVE SIMULATION METHOD FOR UNSTEADY FLOW PREDICTIONS

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1 Introduction

Since the introduction of two-equation models by Kolmogorov (1942) (see Moffat (1991)), they form the foundation of essentially all statistical turbulence models. Two-equation models reflect the basic idea that the minimum information required for modelling the effect of turbulence on the mean flow are two independent scales, obtained from two independent transport equations (e.g. Launder and Spalding, 1974). Two-equation models also form the core of higher order models like full Reynolds Stress models (RSM) (Rotta, 1972, Launder et al., 1975) or Explicit Algebraic Reynolds Stress Models (EARSM) (Pope, 1975, Rodi, 1976, Gatski and Speziale, 1993, Wallin and Johansson, 2000), non-linear stress-strain models (Craft, et al. 1996). Even one-equation models (Baldwin and Barth, 1992, Menter, 1994, 1997, Spalart and Allmaras, 1994) using the eddy-viscosity as a single variable, can be derived from two-equation models using equilibrium assumptions (Menter 1994, 1997).

From a more fundamental standpoint, all the currently used models suffer from lack of an underlying exact transport equation, which can serve as a guide for the model development on a term by term basis. The reason for this deficiency lies in the fact that the exact equation for ε does not describe the large scales, but the dissipative scales. The goal of a two-equation model is however the modelling of the influence of the large scale motions on the mean flow. Due to the lack of an exact equation, the ε- and the ω-equations are modeled in analogy with the equation for the turbulent kinetic energy, k, using purely heuristic arguments. This has several disadvantages. The first is that important terms and physical effects can be missed in the derivation. The second is that additional effects like compressibility, buoyancy, etc. cannot be modeled on an exact basis.

A more consistent approach for formulating a scale-equation has been developed by Rotta (1968, 1972). Instead of using purely heuristic and dimensional arguments, Rotta formulated an exact transport equation for kL, where L is an integral length-scale of turbulence and k is the turbulent kinetic energy. Rotta's equation represents the large scales of turbulence and can therefore serve as a basis for term-by-term modelling. The distinguishing factor of the model proposed by Rotta was the appearance of a natural length-scale in the source terms of the kL-equation, involving a higher derivative of the velocity field. This resulted from the analysis of one of the terms in the exact equation. The availability of a natural length-scale is an attractive feature, because it allows a more subtle reaction of the model to resolved flow features. However, the third derivative proposed by Rotta turned out to be problematic and was never actually used in any of the kL-variants. There are several reasons for the omission of this term. The most important is that it is not intuitively clear, why the third derivative should be more relevant than the second derivative in determining the natural length-scale. In addition, a third derivative is a tedious quantity to compute in a general-purpose CFD code and can easily result in numerical instabilities. With the omission of the higher derivative term, the k-kL-

model lost its main distinguishing factor over the k-ε and the k-ω models. Actually, without this term it proved inferior, as it could not be calibrated for the logarithmic profile without additional ad-hoc terms depending on wall distance. This deficiency has eventually led to the advent of the k-ε model as the major industrial two-equation model. The following quote from Rodi (2004) stresses that this was largely due to the *k-kL-* models inability to handle the log-layer: "In the late 60's and early 70's, some investigations have been carried out at Imperial College, London, with Rotta's *kL* equation, but this requires an extra term near the wall to conform with the log law. Hence in the early 70's there was soon a switch over to the k-ε model … By choosing the constants properly, the (k-ε) model can be made consistent with the log law."

In recent years, steps of modernizing the *kL*-equation have been taken (Menter and Egorov 2004, 2005a, 2005b, 2005c). It is argued that Rotta's assumptions, leading to the term with the third derivative of the velocity field in his *kL*-equation is not consistent with the nature of the underlying term in the exact equation. As a result, the second derivative appears in the model, satisfying the log-law without the need for additional terms. Furthermore, the model lends itself much easier to the introduction of robust low-Re (viscous sublayer model) terms than the k-ε model (Menter at al., 2006). The new model has been re-formulated as a one- and a two-equation model using $\Phi = \sqrt{k}L$ as the new scaling variable in Menter at al., (2006). While the resulting KSKL ($k-\sqrt{k}L$) and SKL ($\sqrt{k}L$) offer interesting alternatives to existing steady RANS models, the more important aspect is their ability of resolving unsteady turbulent structures similar to the behavior of Detached Eddy Simulation (DES) models (e.g. Spalart et al. 1997, Strelets 2001), but without an explicit influence of the grid spacing on the RANS part of the model. As a result, the SAS approach is suitable for the computation of source terms in certain aero-acoustics simulations, with a representation of the turbulent spectrum down to the grid limit.

The behavior of the model in unsteady flow simulations will be the central subject of this article. It will clarify some of the underlying assumptions in the model derivation and discuss the concept of Unsteady RANS (URANS) in light of the Scale-Adaptive Simulation (SAS) characteristics of the KSKL model. It will be shown that the classical URANS behavior of current turbulence models is not a result of the averaging procedure applied to the equations, as widely thought in the community, but of the specific way RANS models have been formulated in the past. This opens the way for using such advanced URANS concepts as a basis for acoustics simulations.

2 Rotta's k-kL Model

2.1 Basic Formulation

Almost all two-equation models use the equation for the turbulent kinetic energy, *k,* for determining one of the two independent large scales of turbulence. The principal unknown term in the *k*-equation is the turbulent dissipation rate, ε which has to be obtained from another transport equation. While an exact equation for ε can be derived, it is not compatible with the need of describing the large scales of turbulence, as the dissipation takes place at the smallest scales of the turbulent spectrum. Similar arguments are true for ω, which is sometimes interpreted as the rate of dissipation per unit of turbulence kinetic energy (see