Immersed Boundary Technique for Large–Eddy–Simulation

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where

$$\tilde{\rho}''_{p_t} = \frac{\tilde{\rho}'_{p_t} + \frac{2}{3}\frac{\Delta\tau}{\Delta t}\tilde{\rho}_{p_t}}{1 + \frac{2}{3}\frac{\Delta\tau}{\Delta t}}. \tag{107}$$

Because $\mathbf{S}$ has the same form as $\mathbf{\Gamma}$ and $\mathbf{P}$, the eigenvectors and eigenvalues also take the same form, yielding an easily computed set of real eigenvalues and eigenvectors.
Now, the scheme is obtained firstly neglecting the non-orthogonal viscous coefficient matrices, $\mathbf{R}_{xy}$, $\mathbf{R}_{xz}$, $\mathbf{R}_{yx}$,$\mathbf{R}_{yz}$,$\mathbf{R}_{zx}$ and $\mathbf{R}_{zy}$, and then factoring out the matrix $\mathbf{S}$ from the LHS:

$$\mathbf{S}\left[\mathbf{I} + \Delta\tau\mathbf{S}^{-1}\frac{\partial}{\partial x}\left(\mathbf{A}_v - \mathbf{R}_{xx}\frac{\partial}{\partial x}\right) + \Delta\tau\mathbf{S}^{-1}\frac{\partial}{\partial y}\left(\mathbf{B}_v - \mathbf{R}_{yy}\frac{\partial}{\partial y}\right)\right.$$
$$\left. + \Delta\tau\mathbf{S}^{-1}\frac{\partial}{\partial z}\left(\mathbf{C}_v - \mathbf{R}_{zz}\frac{\partial}{\partial z}\right)\right]\Delta Q_v = -\Delta\tau\left[\frac{3Q^{n+1,m} - 4Q^n + Q^{n-1}}{2\Delta t} + \mathcal{R}^{n+1,m}\right]. \tag{108}$$

Diagonalizing following the procedure by Pulliam and Chaussee (69), the matrices $\mathbf{S}^{-1}\mathbf{A}_v$, $\mathbf{S}^{-1}\mathbf{B}_v$ and $\mathbf{S}^{-1}\mathbf{C}_v$ can be written as:

$$\mathbf{S}^{-1}\mathbf{A}_v = \mathbf{M}_x\mathbf{\Lambda}_x\mathbf{M}_x^{-1}. \tag{109}$$

$$\mathbf{S}^{-1}\mathbf{B}_v = \mathbf{M}_y\mathbf{\Lambda}_y\mathbf{M}_y^{-1}. \tag{110}$$

$$\mathbf{S}^{-1}\mathbf{C}_v = \mathbf{M}_z\mathbf{\Lambda}_z\mathbf{M}_z^{-1}. \tag{111}$$

For the inviscid equations, the eigenvector matrices can be factored out of the implicit operator. The viscous terms, however, present another difficulty. The linearized viscous coefficient matrix $\mathbf{R}_{ij}$ is not diagonalized by the same eigenvectors as the inviscid flux terms. The current method for incorporating viscous effects into the LHS is to replace the viscous coefficient matrix with its spectral radius R_s times the identity matrix. The spectral radius is defined as below (Schwer (80)):

$$R_s = \max\left(\frac{\mu + \mu_t}{\rho}, \frac{\tilde{\rho}''_{p_t} K}{\rho\tilde{\rho}''_{p_t}\tilde{h}_T + \tilde{\rho}_T(1 - \rho\tilde{h}_{p_t})}\right). \tag{112}$$

The diagonalized scheme is then written as:

$$\mathbf{S}\left[\mathbf{I} + \Delta\tau\mathbf{M}_x\frac{\partial}{\partial x}\left(\mathbf{\Lambda}_x - R_x\mathbf{I}\frac{\partial}{\partial x}\right)\mathbf{M}_x^{-1} + \Delta\tau\mathbf{M}_y\frac{\partial}{\partial y}\left(\mathbf{\Lambda}_y - R_y\mathbf{I}\frac{\partial}{\partial y}\right)\mathbf{M}_y^{-1}\right.$$
$$\left. + \Delta\tau\mathbf{M}_z\frac{\partial}{\partial z}\left(\mathbf{\Lambda}_z - R_z\mathbf{I}\frac{\partial}{\partial z}\right)\mathbf{M}_z^{-1}\right]\Delta Q_v = -\Delta\tau\left[\frac{3Q^{n+1,m} - 4Q^n + Q^{n-1}}{2\Delta t} + \mathcal{R}^{n+1,m}\right]. \tag{113}$$

One potential drawback to the above method for diagonalizing the LHS is that the modal analysis (based on $\mathbf{S}^{-1}\mathbf{A}_v$) appears to be inconsistent with that used for the RHS dissipation terms, based on $\mathbf{\Gamma}^{-1}\mathbf{A}_v$) (see Subsection A.19). Due to the nature of the preconditioning, this does not appear to be a problem. Examining Equation (106) and Equation (101), the only difference between $\mathbf{S}$ and $\mathbf{\Gamma}$ is a scalar multiplier and the definition of $\tilde{\rho}''_{p_t}$ found in Equation (107). Since preconditioning is employed under low Mach number conditions, there are primarily two limiting cases to examine: 1) temporal resolution of acoustic