Shape Optimization for Fluids

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Abstract

The numerical tools for Optimal Shape Design (OSD) derive from Optimization theory. Differentiable Optimization is based on gradient methods and when the gradients are difficult to compute then non gradient based methods are preferred such as topological optimization and genetic algorithms. In these talks we will survey the fundamentals, algorithms and results of gradient based methods and introduce the concepts and present some results from topological optimization. Most of the applications will be taken from compressible fluid dynamics (Euler and Navier-Stokes equations) for aerospace. In the last part we will also discuss the specific problems associated with shocks. Mesh adaptivity and automatic differentiation (AD) of computer program are used extensively; therefore they will also be discussed in these two talks.

keywords: Optimization, shape design, , adaptation, aerodynamics, micro-fluidic, topological optimization.

1 Introduction

Hadamard (1910) may have been the first applied mathematician to derive a formula for the sensitivity of a Partial Differential Equation (PDE) with respect to the shape of its domain. This opened the field of Optimal Shape Design (OSD). But the field as we know it now really began in the seventies as an offspring of optimal control theory for distributed systems (Lions (1968)) and the calculus of variation. So OSD has borrowed the vocabulary of control theory: the design is done by minimizing a criteria, which depends upon a state variable, solution of set of partial differential equations (PDE) and function of a control variable, the shape.
Among others, Cea et al (1973), Pironneau (1973), Murat-Simon (1976), Cea (in Haug et al (1978) gave methods to derive optimality conditions for the continuous problems and Begis et al (1976) Morice (1976) and Marrocco et al (1978) for the discretized problems, in the same French school of applied mathematics at IRIA (now INRIA) and the University of Paris VI, both headed by Jacques-Louis Lions. Theoretical results on existence of solutions were obtained by Chenais (1975), Sverak (1992) Bucur et al. (1995) and Liu et al (1999); a counter example to existence was produced by Tartar (1975) in a key paper which linked optimal shape design with homogenization theory in what is now known as "topological optimization".

Most design engineers do their optimization by hand, intuitively, but up to a certain point! It is common knowledge that if the number of parameters $P$ is greater than 4 or so, intuition fails. There are also commercial packages which find the minimum of a functional with respect to parameters and require from the user only a subroutine to evaluate the cost function for a given design. These packages are usually based on local variation methods (Powell(1970)), involving polynomial fits of the functional from point evaluations. They are expensive here because they require $O(P^2)$ solutions of the state equations (e.g. the flow solver). However, recently Powell came up with a new algorithm, called NEWOA, which beats many of the genetic packages and is certainly the best "quick and dirty" solution to an optimal shape problem when the number of design parameters are small.

But for 3D wings for example, there are hundreds of design parameters so that shape optimization requires a complete numerical treatment with a robust differentiable optimization package and a precise sensitivity analysis with respect to the shape of the wing.

A numerical fluid solver can be viewed as a C function with an input and an output, the design variables which define the wing shape and the drag for instance. Sensitivity analysis finds the gradient of the cost function with respect to the design variables. It is difficult when the fluid is compressible. An alternative is to let the computer do it for you by using a software for "Automatic Differentiation of programs" such as ADOL-C. This approach is extremely convenient and we shall give here a brief presentation. But to understand it fully it is better to know the analytical approach as well; this is the object of the paragraph on sensitivity analysis. More details can be found in Pironneau (1983), Neittanmaki (1991), and Banichuk (1990).