INTRODUCTION TO THE EFFECTS OF ROTATION ON TURBULENCE

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1. Preliminaries

Rotating flows are encountered in astrophysical, geophysical and engineering fluid mechanics. Research on rotating flows in geophysical settings is undertaken with the view to enhance our understanding of the motion of air in the atmosphere and the water currents in the oceans and fjords, which in turn is essential for transport processes and dispersion of pollutants in our environment. In man-made rotating devices, e.g. turbines, pumps, and compressors, system rotation is part of the design concept and the performance depends on the rate of rotation.

A common and noteworthy feature is that rotating flows are almost always *turbulent* like the natural atmospheric boundary layers and ocean currents as well as the engineering flows in rotating machinery. Such flows are significantly affected by rotation and both the rate of rotation and the orientation of the axis of rotation relative to the flow direction matter. The characteristic length- and time-scales may differ by several orders of magnitude from a geophysical to an industrial flow problem. For this reason alone, some characteristic rotational effects encountered in natural flows are not occurring in engineering devices. For example, the effects of rotation on the turbulence in geophysical flows are often of minor importance since the turbulent scales are substantially smaller than the scales of the main stream.

Let us from the very beginning make it clear that this lecture is concerned with fluid flow in rotating systems or environments. The focus is on flow phenomena which only arise when a flow system is rotated. In view of the scope of the Lecture Series, the aim is to give an overview of the rotational-induced effects which have relevance for turbomachinery flows. The effect of rotation both on the mean fluid motion and on the turbulence will be discussed in some detail since they are strongly inter-coupled.

In the exposition to follow a number of simplifying assumptions will be made:

- Incompressible flow. We consider the flow to be incompressible, i.e. that the density of the fluid will not vary as a result of the motion. Flows of gases are therefore not excluded as long as the density variations are of negligible importance.
- Unstratified flow. Stratification caused by variations in temperature or salinity is of major importance in geophysical fluid mechanics but are not considered here.

- Constant rotation speed. The system rotation is assumed to have a constant magnitude. This assumption excludes start-up or shut-down processes.
- Fixed rotation axis. Atmospheric motions are mostly affected by the component of the Coriolis force tangential to the surface of the planet, e.g. the Earth. This component is proportional to $\sin\phi$, where ϕ is the local latitude. Geophysical flow phenomena caused by the variability of ϕ along the Earth's surface are of no relevance in turbomachinery and other industrial applications.
- Body forces like gravity, magnetic and electric body forces are not considered although they are often important, for instance in astrophysical problems.

For supplementary reading the classical book by Greenspan (1968) gives a general introduction to rotating flows, whereas chapter 7 in Greitzer (2004) is focused on internal flows. The review article by Johnston (1998) deals with turbulent flows related to turbomachinery applications.

2. Kinematics

In the vast majority engineering applications a dynamical problem is formulated and analyzed in an *inertial* reference frame Rf*. In some cases, however, the problem is more conveniently expressed in a *rotating* reference frame Rf fixed, for instance, to a rotating object or device. A notable daily-life example is the meteorologist who forecasts the motion of weather systems relative to the rotating Earth.

Let us for the sake of generality consider a dynamical problem in the frame Rf which moves with a velocity $\vec{v}_o$ and accelerates with $\vec{a}_o$ relative to the inertial reference frame Rf*, as illustrated in Figure 1. A velocity $\vec{v}^*$ in the inertial reference frame Rf* is related to the velocity $\vec{v}$ at a given place $\vec{r}$ in the moving reference frame Rf according to:

$$\vec{v}^* = \underbrace{\vec{v}_o + \vec{\Omega}\times\vec{r}}_{place\ velocity} + \vec{v}\,. \tag{1}$$

Here, $\vec{\Omega}$ is the angular velocity vector which gives the rotation rate and direction of Rf. The overall relative motion between Rf* and Rf is made up of the translational motion $\vec{v}_o$ and the rotational motion $\vec{\Omega}$ of the place $\vec{r}$, which therefore collectively are called the *place velocity.* Similarly, the time rate-of-change of $\vec{v}^*$, i.e. $\vec{a}^*$, can be expressed as:

$$\dot{\vec{v}}^* = \vec{a}^* = \underbrace{\vec{a}_o + \dot{\vec{\Omega}}\times\vec{r} + \vec{\Omega}\times(\vec{\Omega}\times\vec{r})}_{place\ acceleration} + \underbrace{2\vec{\Omega}\times\vec{v}}_{Coriolis\ acceleration} + \vec{a}\,, \tag{2}$$

where the dot ($\dot{}$) is introduced to denote differentiation with respect to time. Here, $\vec{a}^*$ is the acceleration in Rf* whereas $\vec{a}$ is the acceleration as seen from Rf. Equation (2) thus provides the essential linkage between the acceleration in the two different reference frames.