

# COMPUTATIONAL SIMULATION OF DIRECT CURRENT AND DIELECTRIC BARRIER DISCHARGES

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## 1. Governing Equations

To describe the electromagnetic perturbation, the gas discharge model based on the drift-diffusion theory is adopted. The phenomenological models for simulating plasma flow control has been reviewed by Huang et al.<sup>14</sup> Surzhikov and Shang<sup>41</sup> successfully developed a three equation model of a three-component (neutral, electron, and ion) and two-temperature, weakly ionized air. In their formulation, a transverse externally applied magnetic field is also included. This model has been widely used for flow control using DCD or DBD<sup>4,13,23,31,32,33,34,35</sup>. In self-sustain plasma, the rate of change for the charged particle number density in a control volume must be balanced by generation through ionization and depletion by recombination. The species continuity equations for electron and ion are given by the drift-diffusion formulation<sup>25,41</sup>:

$$\begin{aligned}\frac{\partial n_e}{\partial t} + \vec{\nabla} \cdot \vec{\Gamma}_e &= \alpha(|\vec{E}|, p) |\vec{\Gamma}_e| - \beta n_+ n_e \\ \frac{\partial n_+}{\partial t} + \vec{\nabla} \cdot \vec{\Gamma}_+ &= \alpha(|\vec{E}|, p) |\vec{\Gamma}_e| - \beta n_+ n_e\end{aligned}\quad (1)$$

where

$$\begin{aligned}\vec{\Gamma}_e &= n_e \vec{u}_e - \mu_e n_e (\vec{E} + \vec{u}_e \times \vec{B}) - D_e \vec{\nabla} \cdot n_e \\ \vec{\Gamma}_+ &= n_+ \vec{u}_+ + \mu_+ n_+ (\vec{E} + \vec{u}_+ \times \vec{B}) - D_+ \vec{\nabla} \cdot n_+\end{aligned}\quad (2)$$

In the above formulation,  $\alpha(|\vec{E}|, p)$  and  $\beta$  are the first Townsend ionization coefficient and recombination coefficient, respectively. The parameters  $\mu_e$  and  $\mu_+$  are the electron and ion mobility, and  $D_e$  and  $D_+$  are the electron and ion diffusion coefficients<sup>26</sup>. When the applied transverse magnetic field is aligned with the  $z$  coordinate,  $\vec{B} = B_z \vec{k}$ , the Lorentz acceleration degenerates into two components in the  $x$  and  $y$  coordinates,

$$\begin{aligned}\vec{u}_e \times \vec{B} &= u_{e,y} B_z \vec{i} - u_{e,x} B_z \vec{j} \\ \vec{u}_+ \times \vec{B} &= u_{+,y} B_z \vec{i} - u_{+,x} B_z \vec{j}\end{aligned}\quad (3)$$

A compact formulation is achieved by introducing the Hall parameter for electron and ion as  $\beta_e = \mu_e B_z$  and  $\beta_+ = \mu_+ B_z$ , which are simply the ratios of the cyclotron and averaged charged-and-neutral collision frequency<sup>11,25</sup>. In the presence of a transverse magnetic field of  $B_z$ , only the motions of charge particles in the plane that perpendicular to the applied magnetic field are affected and the elements of the electron and ion flux density vectors can be expressed as:

$$\begin{aligned}
\Gamma_{e,x} &= -\mu_e n_e E_{e,x} - \frac{D_e}{1+\beta_e^2} \frac{\partial n_e}{\partial x} + \frac{\beta_e D_e}{1+\beta_e^2} \frac{\partial n_e}{\partial y} \\
\Gamma_{e,y} &= -\mu_e n_e E_{e,y} - \frac{D_e}{1+\beta_e^2} \frac{\partial n_e}{\partial y} + \frac{\beta_e D_e}{1+\beta_e^2} \frac{\partial n_e}{\partial x} \\
\Gamma_{e,z} &= -\mu_e n_e E_{e,z} - D_e \frac{\partial n_e}{\partial z}
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
\Gamma_{+,x} &= \mu_+ n_+ E_{+,x} - \frac{D_+}{1+\beta_+^2} \frac{\partial n_+}{\partial x} + \frac{\beta_+ D_+}{1+\beta_+^2} \frac{\partial n_+}{\partial y} \\
\Gamma_{+,y} &= \mu_+ n_+ E_{+,y} - \frac{D_+}{1+\beta_+^2} \frac{\partial n_+}{\partial y} + \frac{\beta_+ D_+}{1+\beta_+^2} \frac{\partial n_+}{\partial x} \\
\Gamma_{+,z} &= \mu_+ n_+ E_{+,z} - D_+ \frac{\partial n_+}{\partial z}.
\end{aligned} \tag{5}$$

In the presence of the transverse magnetic field  $B_z$ , the  $z$ -component electric field intensities are unaltered. Thus the effective electric fields of the electrons and positively charged ion can be defined as the following:

$$\begin{aligned}
E_{e,x} &= \frac{E_x - \beta_e E_y}{1+\beta_e^2} \\
E_{e,y} &= \frac{E_y - \beta_e E_x}{1+\beta_e^2}
\end{aligned} \tag{6}$$

and

$$\begin{aligned}
E_{+,x} &= \frac{E_x + \beta_+ E_y}{1+\beta_+^2} \\
E_{+,y} &= \frac{E_y + \beta_+ E_x}{1+\beta_+^2}
\end{aligned} \tag{7}$$

The model equations fully satisfy the generalized Ampere's circuit law and the Gauss's law for electric field<sup>15</sup>:

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \vec{J} = 0 \tag{8}$$

where  $\rho_e = e(n_+ - n_e)$  is the space charge density and the electrical current density is defined as  $\vec{J} = e(\vec{\Gamma}_+ - \vec{\Gamma}_e)$ .

A compatible electrical field intensity,  $\vec{E}$ , of the discharge domain is obtained by satisfying the Gauss' law for electric field.

$$\nabla \cdot (\epsilon \vec{E}) = \rho_c \tag{9}$$

This equation can be further simplified in globally neutral plasma by introducing an electrical potential function,  $\vec{E} = -\nabla \Phi$ . The electrical field potential is then the solution of the well-known Poisson equation of plasmadynamics associated with the net space charge density,  $\rho_c$ .

$$\nabla \cdot (\epsilon \nabla \Phi) = -\rho_c \tag{10}$$

or