CHAPTER I

REMARKS ON THE PHYSICAL BEHAVIOUR OF TURBULENT FLOWS WITH A VIEW TO COMPUTING PROCESSES

I. SOME GENERAL ASPECTS OF TURBULENT MECHANISM

When dealing with turbulent flows, the main difficulties are obviously connected with the nature of Navier Stokes equations. For incompressible flows, they are :

(1.1.) $$\frac{\partial \vec{U}(\vec{x},t)}{\partial t} + \left[\vec{U}(\vec{x},t) . \nabla\right] \vec{U}(\vec{x},t) = - \nabla \frac{P(\vec{x},t)}{\rho} + \nu \nabla^2 \vec{U}($$

in which $\vec{U}(x,t)$ is the three dimensional velocity field, $P(x,t)$ represents the pressure and ν the kinematic viscosity. The fluid is supposed incompressible but even if the Mach number of the turbulent flow is high enough, the mechanism of turbulence is supposed not to be too much affected as far as computing processes are concerned. First, we shall roughly examine why any further difficulties come into play.

The previous equation must be supplemented by the continuity equation :

(1.2.) $$\vec{\nabla} . \vec{U}(\vec{x},t) = 0$$

and by the boundary conditions on the material interface S . On rigid stationary walls, these last conditions are :

$$\vec{U}(\vec{x},t) = 0 \qquad \{\forall \vec{x} \in S\}$$

The pressure field is linked to the velocity field by means of the Poisson equation, by taking the divergence of (1.1.) and applying (1.2.), it follows that :

$$\nabla^2 \frac{P}{\rho} = -\nabla.\left\{(\vec{U}.\nabla)\vec{U}\right\}$$

It is not surprising that the pressure is determined at each instant by the global velocity field if we bear in mind that incompressibility corresponds to the infinite sound propagation speed.

From the two previous relations, it follows that :

$$\frac{d}{dt}\frac{1}{2}\int_{\mathcal{V}} U_i U_i \, d\mathcal{V} = -\nu \int_{\mathcal{V}} \left[\frac{\partial U_i}{\partial x_j} \cdot \frac{\partial U_i}{\partial x_j}\right] d\mathcal{V}$$

where $\mathcal{V}$ is the whole volume occupied by the fluid. If we consider any point in the fluid, the energy balance is obviously much more complicated, for instance, the convective term $(\vec{U}.\nabla)\vec{U}$ carries energy from one point to another. The overall energy balance only throws into light that in the absence of external forces kinetic energy is dissipated by the viscous effect.

So far the mathematical theory of the Navier Stokes equations is incomplete, that is to say that there is no general existence and uniqueness theorem which shows that (1.1.) and (1.2.) associated with suitable boundary conditions are well posed. Only partial results due to LERAY, LADYZHEWSKAYA ... are available. In brief, these results established the existence and uniqueness of solutions in three dimensions

for short times, with arbitrary ν and for all times if ν is large enough. The difficulties encountered for three dimensional flows are avoidable only where two dimensional spaces are concerned ; a fact in agreement with the well known property of differential equations : the mathematical regularity of the solutions depends on the number of dimensions of the mathematical space (1). Some curious results were pointed out by LADYZHEWSKAYA when examining the role of the viscous term (the term of higher degree in the equation). For instance, if a biharmonic damping $\alpha \nabla^4 U$ is added, then the existence and uniqueness of the solution is ended for all α ; under a suitable hypothesis LADYZHEWSKAYA suggested that a satisfactory existence and uniqueness theorem can be proved when the constant viscosity appearing in (1.1.) depends on the deformation rate in a convenient way.

These first remarks emphasize that the mathematical basis of the Navier Stokes equations requires much further investigations ; and, that future mathematical work could act significantly on our field of research. Nowadays, it is not surprising how difficult the problem of turbulence is.

In order to better understand the inertial behaviour of turbulent motions, it is possible to examine the following five mode dynamical system :

(1). Perhaps this mathematical property presents some connections with the three dimensional character of the turbulent motion. The stretching process of vortex lines encountered here is so specific that the scalar product $\vec{U}.\mathrm{rot}\,\vec{U}$ is sometimes proposed in order to classify some complex motions. Complex situations may occur, for instance when the larger scale flow patterns in the atmosphere are closely two dimensional perhaps it is better not to regard them as turbulence though they provide smaller scales of atmospheric motion with energy.