THE RELATIONSHIP BETWEEN A WIENER-HERMITE EXPANSION AND AN ENERGY CASCADE

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ABSTRACT

Meecham and his co-workers have developed a theory of Burgers turbulence involving a truncated Wiener-Hermite expansion of the velocity field. The theory reduces to a set of coupled differential equations for deterministic kernels, the randomness being taken up at the outset by a white-noise function associated with the initial state of the flow. Numerical experiments are reported, in this paper, showing that the theory predicts an insufficient rate of energy decay for Reynolds numbers larger than two. The reason is found to be that the equations for the kernels contain no convolution integrals in wave-number space and therefore permit no cascade of energy. It is shown that an energy cascade in wave-number space corresponds to a cascade of energy up through successive terms of the Wiener-Hermite expansion. Numerical experiments on Burgers equation itself are carried out, the first Wiener-Hermite kernel is extracted and the Gaussian and non-Gaussian components of an actual Burgers solution are presented. The figures show directly that only higher-order terms in the Wiener-Hermite expansion are capable of representing a shock structure and confirm the result that higher-order terms are associated with high wave numbers. The Wiener-Hermite expansion is examined from a fundamental point of view, and the conclusion is reached that "Gaussianity," in the

experimentalist's sense, has no bearing whatever on the rate of convergence of an expansion whose white-noise function is associated with the initial state. Such an expansion would converge rapidly only if the velocity field and its initial state were joint-normally distributed.