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1. FUNDAMENTAL ASPECTS OF BOUNDARY LAYERS WITH LONGITUDINAL CONCAVE CURVATURE

1.1 Introduction

A boundary layer developing on a concave surface is inherently unstable because the centre of curvature lies in the direction of increasing mean velocity. A simplified explanation is as follows. Consider the random displacement of a "lump" of fluid away from the wall, as indicated in Fig. 1.1 It will carry with it some "memory" of its original mean velocity, so that its velocity in its new position will be less than that of the local fluid. The force F_P exerted on the "lump" by the radial pressure gradient then exceeds the centrifugal force F_C so it will be carried further from the wall. If a similar displacement occurs over a convex wall, both F_P and F_C are reversed in direction but the "lump" still retains a lower velocity than the local fluid, so the imbalance between F_P and F_C is now a restoring force, towards the wall. Thus this form of instability exists only for walls with longitudinal concave curvature, although analogous forms, *e.g.* caused by heating the wall from below, can exist on flat and convex walls.

If the instability cannot be damped by viscosity, small disturbances will grow and eventually develop into a system of contra-rotating vortices (with vortex axes in the longitudinal direction), first predicted by Görtler (1940) and named after him. Numerous experiments in laminar layers have demonstrated the existence of these vortices, which can affect the boundary layer growth rate, the skin friction and heat transfer, and can interact with freestream turbulence and Tollmien-Schlichting waves to affect transition. What happens to turbulent layers depends on whether transition or the start of concave wall curvature occurs first. There is evidence that pre-existing Görtler vortices can persist through transition to affect the developing turbulent layer, but if a turbulent layer is established on a flat or convex wall upstream of the concave wall, distinct longitudinal vortices (or "roll cells") do not always appear. The principal effect of concave curvature on a turbulent layer is an increase in mixing, giving rise to increased skin friction and heat transfer; a corresponding decrease occurs with convex curvature.

1.2 Boundary layer equations and some basic relationships

The two-dimensional, incompressible, laminar boundary layer equations, with constant viscosity, can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial}{\partial y}\left\{(1 + ky)v\right\} = 0 \tag{1.1}$$

$$u\frac{\partial u}{\partial x} + (1 + ky)\, v\frac{\partial u}{\partial y} + kuv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + (1 + ky)\nu \frac{\partial^2 u}{\partial y^2} + \nu k \left\{\frac{\partial u}{\partial y} - \frac{ku}{1+ky}\right\} \tag{1.2}$$