VKI-LECTURE SERIES ON MODELLING OF COMBUSTION AND TURBULENCE

March, 9-13, 1992

Pdf Transport Modelling

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SUMMARY.

Probability density functions (pdfs) provide a complete statistical description of turbulent flows at a single point or a finite number of points. Their main application is the prediction of turbulent reacting flows. The lectures are organized as follows:

(1) Basic theory.

The properties of pdfs and the characteristic function are reviewed and the pdf equation is derived from first principles. The hierarchy of pdf equations is discussed and the relation of moment equations to the pdf equation is established. All processes affecting the evolution of the pdf appear in the pdf equation as terms with distinct properties such as diffusive, destructive or productive. These properties will be discussed in detail to prepare the ground for the closure models to developed later. The relation of the pdf equation to stochastic processes and stochastic fields will be established covering Markov processes and the Fokker-Planck equation.

(2) Closure of the pdf equation.

The single point pdf equation is considered as vehicle for the development of closures. The closure of the turbulent flux term in the pdf equation will be discussed and compared to the closure for the joint velocity-scalar pdf equation where turbulent diffusion is closed but the pressure gradient requires a closure model. The main part of this chapter will be devoted to mixing models for reacting turbulent flows. Mixing models based on simplified notion on the dynamics of the mixing process and models based on mapping procedures will be developed and analyzed in detail.

(3) Computational methods.

The numerical solution of moment equations and the pdf equation will be outlined briefly. Finite-difference methods are considered for moment equations and stochastic simulation techniques for the pdf equation.

(4) Applications.

Pdf methods have been applied to a variety of reacting and nonreacting turbulent flows. The simulation of combustion in maintained homogeneous turbulence and several turbulent nonpremixed flames will be presented and the results will be evaluated by comparison with available experimental data.

(5) Conclusions.

PDF TRANSPORT MODELLING

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(1) Basic theory.

The recent past has seen significant progress in the theoretical development and application of transport equations for probability density functions (pdfs) to turbulent flows with chemical reactions (see Pope (1985), Borghi (1988), Bilger (1989), Kollmann (1990)). Pdf methods derive their justification from the basic fact, that turbulent convection and chemical reactions can be dealt with in exact and closed form. This is in stark contrast to the approach based on statistical moments, which requires closure models for nonlinear processes such as convection or chemical reactions. Pdf methods succeed here, because they transform certain nonlinear processes into linear terms with variable coefficients by converting the associated dependent variables in the basic laws into independent variables of the pdf. Hence, two of the most important closure problems encountered in moment equations are overcome by pdf methods. Furthermore, pdfs provide a complete statistical description of the fluctuations at a single point or a finite number of points in the flow field. However, it will be shown that the equation governing the evolution of the pdf at a single points is indeterminate, because the terms accounting for molecular transport and the fluctuating pressure-gradient require the pdf at two points. The closure problem for these two terms must be overcome to arrive at a determinate equation. The development of closure models requires the understanding of the pdf equation and the processes governing its evolution. Therefore, we turn first to a discussion of the basic properties of pdfs and their transport equations.

(1.1) Properties of pdfs

Consider a random vector $\underline{Y}$ of dimension $n > 1$, then we define the cumulative distribution function F_n by

$$F_n(y_1, \cdots, y_n) \equiv \mu(\{-\infty < Y_\alpha \leq y_\alpha, \alpha = 1, \cdots, n\}) \tag{1}$$

where μ denotes the probability measure defined on the σ-algebra $\mathcal{A}$ of events which can be set up as at most countably many combinations (union and intersection) of elementary events taken from the sample space $\Omega \subseteq R^n$. It has the properties

$$\lim_{y_\alpha \to -\infty} F_n(y_1, \cdots, y_n) = 0 \tag{2}$$

for any $1 \leq \alpha \leq n$, and

$$\lim_{y_1, \cdots, y_n \to \infty} F_n(y_1, \cdots, y_n) = 1 \tag{3}$$

and the cdf (cumulative distribution function) F_n is a monotonically nondecreasing function of its arguments. It has the reduction property that the lower dimensional cdfs can be established by setting independent variables to ∞. For instance, the cdf F_{n-1} follows from F_n in the form

$$F_{n-1}(y_1, \cdots, y_{n-1}) = \lim_{y_n \to \infty} F_n(y_1, \cdots, y_n)$$