

INTRODUCTION

Classical or continuum fluid mechanics ignores the fact that a fluid is actually a collection of interacting molecules. From this fact alone it would appear obvious that the treatment of fluid flow problems from the continuum point of view must be valid only in some limiting sense. It will be the objective of this lecture to show that a more fundamental treatment of a certain class of fluid-mechanics problems is possible. The main features of this exposition will be:

a) A review of the underlying features and assumptions of the kinetic theory applied to a dilute gas.

b) A demonstration of the fact that the conservation equations of continuum fluid mechanics (the so-called Navier-Stokes equation) are in fact simply a limiting case of the fundamental equations of kinetic theory. More importantly, the limits of the validity of the Navier-Stokes equations may be set in a rigorous manner.

c) A discussion of the various techniques now available for treating fluid mechanics problems under conditions such that the Navier-Stokes equations are not a valid representation of the phenomena.

KINETIC THEORY

In this section we present a brief review of the underlying assumptions of the kinetic theory as applied to dilute gases, the relevant equations whose solution will provide the engineer with useful macroscopic information on the state of a dynamic system, and the boundary conditions which must be satisfied before the problem may be considered fully solved.

There are two ways to approach the description of a gas by means of molecular concepts. The first would attempt to write Newton's law for each

particle in the flow field and then solve the resultant set of equations yielding the velocity of all molecules as a function of position and time. Recalling that there are 10^{19} particles in each cubic centimeter of gas at normal temperatures and pressures, it is clear that solving even 10 orders-of-magnitude fewer equations would be a horrendous task for the most sophisticated computer. Supplying the proper initial and boundary conditions for each would, in addition, be impossible; thus a detailed approach such as this will not be considered further.

The obviously reasonable approach to the description of an ensemble of molecules is along statistical lines. A question arises as to whether or not a statistical treatment of the problem is complete or not. We take the point of view here that a statistical description of a gas is complete when everything we might be interested in knowing as engineers; e.g., flow velocity, heat flux, forces, etc.; may be calculated from the solution to an evolution equation for the system plus suitable boundary and initial conditions.

Before discussing in detail the statistical representation of a gas, it is essential to state two limitations imposed on this description. Firstly, the theory is designed only to apply to spherical molecules with no internal structure (point-particles); the molecules which make up a monatomic gas are thus well represented by such a theory, at least at temperatures sufficiently low such that the state of the atom's electronic structure is fixed. This limitation is imposed because it allows one to fully describe the state of such a molecule by giving its translational or kinetic velocity, ξ_i . The resulting theory is already quite complicated; the additional complexity introduced by considering the velocities