Sound Propagation in Rarefied Atmospheres

by J.J. Smolderen

One of the techniques used to determine atmospheric temperature profiles is to detonate grenades at various altitudes and measure the time required for each detonation to propagate over a given distance -- usually to a ground station. An essential assumption is that the atmospheric disturbance produced by a grenade may be treated as a sound wave; i.e., a small disturbance; and that its speed of propagation is given by the inviscid value, $a_0 = \sqrt{\gamma R T_0}$. Our purpose here will be to examine the validity of this assumption and to summarize briefly the present status of research in the general area of sound propagation.

Two conditions must be imposed if one wishes to describe "sound" in a fluid medium :

1) recognizing that sound is actually an unsteady flow process, we nevertheless restrict our definition of sound to a process which involves small disturbances; i.e., $u/a_0 << 1$ (where a_0 is the sound velocity in the medium), $\delta p/p_0 << 1$, $\delta T/T_0 << 1$, etc.)

2) the frequencies involved must be such that the process is truly unsteady and not a quasi or near-steady aerodynamic process.

If we adopt the inviscid or Euler equations as a sufficiently correct description of fluid phenomena, then the above conditions result in the existence of a velocity potential which is found to satisfy the acoustic wave equation.

$$\frac{1}{a_0^2}\frac{\partial^2\phi}{\partial t^2} - \Delta\phi = 0; \qquad \text{(A-1)}$$

i.e., a disturbance will be propagated in all directions, unattenuated, with velocity $a_0 = \sqrt{\gamma R T_0}$. This result is the basis of the conventional theory of acoustics and provides a sufficient description from the continuum point of view as we shall see later, unless one is interested in either attenuation or second-order effects.

The next step in complexity involves the full Navier-Stokes equation. By invoking the small disturbance assumption, the equations can be linearized. Then by assuming that all quantities vary sinusoidally with respect to both time and position, we can write that the velocity, for example, of a little fluid element which has been influenced by the sound wave is :

$$u = Ue^{i(wt + \zeta x)} \qquad \text{(A-2)}$$

where w is the frequency and ζ the propagation constant. If ζ is real, then points of constant phase propagate with a velocity $a \equiv w/\zeta$. If ζ is complex ($\zeta = \zeta_1 + i\,\zeta_2$) then the variation of the fluid properties is, e.g.,

$$u = Ue^{i(wt + \zeta_1 x)} e^{-\zeta_2 x} \qquad \text{(A-3)}$$

where the last factor clearly represents an attenuation term. The amplitude of the wave decreases with distance exponentially and ζ_2 is the logarithmic decrement or attenuation per unit length. Thus the study of sound propagation using the Navier-Stokes equation becomes a problem of finding ζ for a given frequency w and then using the fact that

$$\text{speed of propagation } a = \frac{w}{Re\,\zeta} = \frac{w}{\zeta_1}$$

$$\text{attenuation} = Im\,\zeta = \zeta_2 \qquad \text{(A-4)}$$