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Coupled Blade-Disk-Shroud Flutter Instabilities in Turbojet Engine Rotors

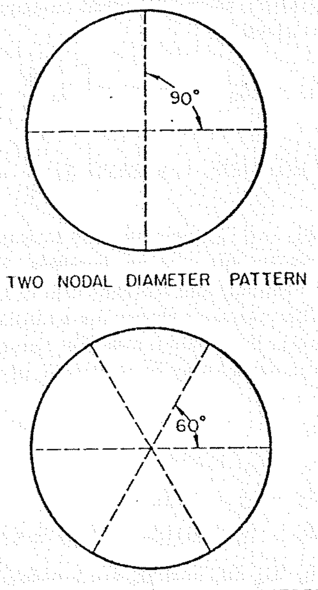
An energy method is used to investigate a flutter instability of turbojet engine rotors which is caused by the interactions between unsteady air loading and the coupled vibration modes of the rotating blade-disk-shroud system. It is shown, analytically, in this parametric study that under certain circumstances the coupling between blade modes permits the transfer of energy from the air to the blade-disk-shroud system, giving rise to a self-excited instability. Both unsteady potential flow theory and empirical data for oscillating airfoils at high incidence are used.

Introduction

THE VIBRATORY mode shapes which can exist on a rotor consisting of a flexible blade-disk-shroud system are well known to structural dynamicists in the turbomachinery field and a detailed discussion of these modes is beyond the scope and purpose of this paper. Although both concentric and diametric modes can occur, the latter are the only system modes which are of interest in the present paper. These diametric modes are characterized by node lines lying along the diameters of the wheel and having a constant angular spacing. Thus, for example, a two-nodal-diameter mode would have two node lines intersecting normally at the center of the disk, and a three-nodal-diameter mode would have three node lines intersecting at the disk center with an angular spacing of 60 deg between adjacent node lines (see Fig. 1). These diametric modes are the physical embodiment of the eigensolutions of the system, and it can be shown, using standard structural dynamics techniques, that the system frequency for each mode is primarily a function of the physical distribution of the system mass and stiffness and is only slightly affected by the rotation of the system. Thus the system frequencies do not necessarily coincide with integral multiples of the rotor speed, and in fact, such coincidences of frequency are avoided for the lower frequencies if possible.

Recently, a number of instances of nonintegral order vibrations at high stress have occurred in both engine and test rig compressor rotors. The stress levels reached in a number of these cases were sufficiently high to severely limit the safe operating range of the compressor. Attempts to relate these vibrations to the stall flutter phenomenon or to rotating stall have essentially failed, largely because the vibrations often occur on or near the engine operating line. Subsequent analysis of these cases has revealed that the observed frequencies of these instabilities have correlated well with the predicted frequencies of the coupled blade-disk-shroud motion described previously. In view of the available information, it appears likely that the coupled motion can result in a system instability, even under potential flow conditions.

The object of this study is to explore the underlying mechanism of this instability and to show that under certain conditions of airflow and rotor geometry this coupled oscillation is capable of extracting energy from the airstream in sufficient quantities to produce an unstable vibratory motion.



a) TWO NODAL DIAMETER PATTERN

b) THREE NODAL DIAMETER PATTERN

Fig. 1 Typical diametric node configurations

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Nomenclature

a = dimensionless distance of pivot axis aft of the midchord, in semichords
A = lift function
b = semichord, ft
B = moment function
c = damping, lb-sec/ft
c_{cr} = critical damping, lb-sec/ft
C = coefficient
$C(k)$ = Theodorsen function
f = frequency, cps
F_d = damping force, lb
$F(k)$ = real part of Theodorsen function
$G(k)$ = imaginary part of Theodorsen function
h' = bending deflection, ft (positive downward)
h = h'/b = dimensionless bending deflection, in semichords
i = $\sqrt{-1}$
k = $b\omega/U$ = reduced frequency parameter
K = stiffness, lb/ft
K_E = kinetic energy, ft-lb
L = lift, lb (positive upward), or lift function
m = mass, lb-sec²/ft
M = moment, ft-lb (positive nose up), or moment function
n = number of blades
r = radius, ft
s' = peripheral distance along rim, ft
s = s'/S = dimensionless peripheral distance along rim
S = peripheral wave length, ft
t = time, sec
U = velocity relative to moving blade, ft/sec
W = work, ft-lb
x = displacement, ft
α = twist angle, rad, or incidence angle, deg (positive nose up)

(Continued on next page)

Analysis

Two-Dimensional Section Coefficients

The unsteady aerodynamic theory of a two-dimensional thin airfoil executing simple harmonic motion in vertical translation and/or twist has been extensively investigated by a number of authors [1-5][1] and will not be discussed in any great detail in this paper. However, for clarity in the ensuing derivation, it is expedient to describe briefly the physical system being considered and to define the nomenclature to be used in the analysis.

Fig. 2 is a schematic representation of a two-dimensional airfoil section displaced in both vertical translation (normal to the chord) and twist. The effects of translation of the airfoil parallel to the chord are of second order [6] and have been neglected herein. The complex, time-dependent unsteady lift and moment per unit span are given by

$$L = L_R + iL_I = -\pi\rho b^3\omega^2\left[A_h\frac{h'}{b} + A_\alpha\,\alpha\right] \qquad (1)$$

$$M = M_R + iM_I = \pi\rho b^4\omega^2\left[B_h\frac{h'}{b} + B_\alpha\alpha\right] \qquad (2)$$

where A_h, A_α, B_h, and B_α represent the standard unsteady aerodynamic coefficients—lift due to bending, lift due to twist, moment due to bending, and moment due to twist, respectively. For example, if Theodorsen's theory for an isolated airfoil at zero incidence oscillating in an incompressible, two-dimensional flow is used [1], these quantities may be rewritten, in the form of [7], as

$$\left.\begin{aligned} A_h &= L_h \\ A_\alpha &= L_\alpha - (\tfrac{1}{2} + a)\,L_h \\ B_h &= M_h - (\tfrac{1}{2} + a)\,L_h \\ B_\alpha &= M_\alpha - (\tfrac{1}{2} + a)\,(L_\alpha + M_h) + (\tfrac{1}{2} + a)^2 L_h \end{aligned}\right\} \qquad (3)$$

where, in turn, L_h, L_α, M_h, and M_α are tabulated in both [3] and [7]. (Note that in [3] and [7] the positive lift and vertical translation are both directed downward which accounts for the negative right-hand side of equation (1).) Appropriate coefficients for other aerodynamic conditions may be inserted for A_h, A_α, B_h, and B_α; e.g., the unsteady incompressible aerodynamic coefficients for airfoils in cascade at zero incidence may be taken from [6] or [8], and the effects of incidence on an isolated airfoil may be simulated by using the empirical results of [9]. At present, though, the development will be based on the coefficients A_h, A_α, B_h, B_α, and consequently will be quite general.

It is well known from unsteady aerodynamic theory that the forces and moments acting on an oscillating airfoil are not in phase with the motions producing these forces and moments. A convenient representation of this phenomenon is obtained on writing the unsteady coefficients in complex form as $A_h = A_{hR} + iA_{hI}$, etc., and the time dependent displacements as

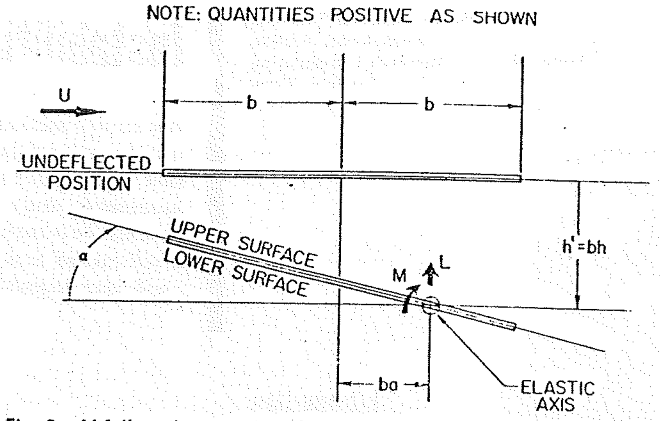

Fig. 2 Airfoil section notation showing both undeflected and deflected blade

$$\left.\begin{aligned} h &= h_R + ih_I = \bar{h}e^{i\omega t} = \bar{h}\cos\omega t + i\bar{h}\sin\omega t \\ \alpha &= \alpha_R + i\alpha_I = \bar{\alpha}e^{i(\omega t+\theta)} = \bar{\alpha}\cos(\omega t + \theta) \\ &\qquad + i\bar{\alpha}\sin(\omega t + \theta) \end{aligned}\right\} \qquad (4)$$

where, in general, it has been assumed that the torsional motion leads the bending motion by a phase angle, θ. In this equation, $h = h'/b$ is the dimensionless bending displacement, and $\bar{h}$ and $\bar{\alpha}$ are the dimensionless amplitudes of the motion in bending and torsion, respectively.

Two-Dimensional Work Per Cycle

The differential work done by the aerodynamic forces and moments in the course of this motion is obtained by computing the product of the in-phase components of force and differential vertical displacement and moment and differential twist. Accordingly, the work done per cycle of motion in each mode is obtained by integrating the differential work in each mode over one cycle. The total work done per cycle of coupled motion is given by the sum

$$W_{TOT} = -b\oint L_R dh_R + \oint M_R d\alpha_R \qquad (5)$$

where the minus sign is required because L and h are defined to be positive in opposite directions. It is important to note that in equation (5), positive work implies instability since these equations represent work done *by* the air forces *on* the system.

To compute these integrals, L_R and M_R are obtained from equations (1) and (2), the real parts of equations (4) are differentiated, and these quantities are substituted into equation (5) to yield

$$\begin{aligned} W_{TOT} = -\pi\rho b^4\omega^2\{\bar{h}\oint[&A_{hR}\bar{h}\cos\omega t - A_{hI}\bar{h}\sin\omega t \\ &+ A_{\alpha R}\bar{\alpha}\cos(\omega t + \theta) - A_{\alpha I}\bar{\alpha}\sin(\omega t + \theta)]\sin\omega t\, d(\omega t) \\ &+ \bar{\alpha}\oint[B_{hR}\bar{h}\cos\omega t - B_{hI}\bar{h}\sin\omega t + B_{\alpha R}\bar{\alpha}\cos(\omega t + \theta) \\ &- B_{\alpha I}\bar{\alpha}\sin(\omega t + \theta)]\sin(\omega t + \theta)d(\omega t)\} \end{aligned} \qquad (6)$$

The line integrals over one cycle of motion are equivalent to an

[1] Numbers in brackets designate References at end of paper.

Nomenclature

- α^* = stalling angle parameter [see equation (30)]
- α_{CH} = chordal stagger angle, deg
- α_s = stalling angle, deg
- γ = damping ratio [see equation (21)]
- δ = logarithmic decrement, or blade deflection, ft
- η = dimensionless spanwise station
- θ = phase angle between bending and torsional motions, rad
- ρ = air density, lb-sec²/ft⁴
- τ = gap, ft
- ϕ = phase angle between force and response
- ω = frequency, rad/sec

Subscripts

- ax = axial
- h = due to bending
- I = imaginary part
- L = lift
- M = moment
- 0 = root radius or natural frequency
- P = due to pitch
- R = real part
- s = stall
- tan = tangential
- T = due to translation or tip value
- TOT = total
- α = due to pitch

Superscripts

- $(\bar{\ })$ = amplitude or average over one cycle
- $(\dot{\ })$, $(\ddot{\ })$ = first and second derivatives with respect to time