Mechanics of arterial blood flow T.J. Pedley 

## 1. Description of the arterial system

The systemic arterial system is a system of branched tubes which decrease peripherally in both area and length. The total cross-sectional area increases, but far less dramatically than in the airways of the lung. Blood is ejected from the left ventricle of the heart, through the aortic valve, into the main artery (the aorta), which begins with a tight curve of more than 180° (the arch) and subsequently tapers noticeably. Other arteries branch off the aorta, large ones to the head and arms (taking about 25% of the blood), medium ones to the ribs (3%), large ones to the kidneys and abdomen (40%), and finally it divides into two to the iliac arteries. Each of these arteries subdivides further, into smaller arteries, and finally into arterioles and capillaries (the microcirculation).

The total volume of blood in the circulation is about 6 L; the volume ejected per minute is about 5 L; the frequency of heart beat is about 70 per min. Thus the volume ejected per beat is about 70 ml. Dimensions, mean velocities, and mean Reynolds numbers in different parts of the circulation are given in Table 1.

The beating of the heart is pulsatile, so the pressure, pressure gradient and flow rate throughout a large part of the system are also pulsatile. In fact, the pressure in all arteries varies between about 80 mm Hg and 120 mm Hg. Although the mean velocity in the aorta is 30 cm/sec, the peak is about 140 cm/sec, and the minimum is about -30 cm/sec (reverse flow).

There is very little change in mean pressure throughout the arterial system, but there is a very steep fall in the arterioles, before a levelling off in the capillaries. The resistance is concentrated in the arterioles. This is functionally sensible: the blood is distributed rapidly, with little energy loss, to all parts of the body; it is then slowed down just before it enters the capillaries where the mass transfer takes place. Furthermore, the pulsatility also dies out in the arterioles, which is also useful to maintain a more or less continuous flow in the capillaries; stopping and starting requires more energy (see lecture on microcirculation).

In a system of rigid tubes, containing an incompressible fluid, the pulsatility would not die out, the pressure everywhere having the same time course as that at the heart (scaled down peripherally). It is the elasticity of the artery walls which, acting as a capacitance, causes the pulsatility to die out by allowing pressure waves to be transmitted with a finite speed. Artery walls consist of an inhomogeneous, anisotropic mixture of substances of various elastic and viscoelastic properties. It is only for very small deformations that they can be considered to be simple elastic solids. The elastic properties of artery wall are dominated by two substances - elastin (Youngs modulus $E \approx 3 \times 10^6$ dynes $cm^{-2}$) and collagen ($E \approx 10^9$), each with a Poisson's ratio of 0.5 (incompressible). Fig. 1 shows how E for artery wall varies with the distension above the unstressed state; a marked jump from a value close to that for elastin towards that for collagen is observed at about the physiological distension. This aids stability, but makes non-linear analysis difficult.

Whole blood is a suspension of blood cells in plasma, which is itself a solution of very large molecules, but can be regarded as a Newtonian fluid at length scales greater than or equal to a capillary radius and at physiological flow rates. In whole blood the cells take up about 45% by volume, and are comprised mostly of red cells; we suppose that white cells and platelets are dynamically negligible. The red cells are biconcave disks of maximum diameter $\approx 9\mu m$, which is the same order of magnitude as a capillary diameter (see lecture on microcirculation). However, in arteries of radius .1 cm or greater, the red cell diameter and spacing are much smaller than the artery radius, and blood can be accurately regarded as a continuum. Also, at physiological shear rates, it is approximately a Newtonian fluid of viscosity about .04 poise. We assume that it is Newtonian.

The sort of fluid mechanical problem we shall try to solve is a prediction of the pressure and flow everywhere in the system, given the pressure P(t) and the flow rate imposed at the heart. Observations of the pressure and flow-rate have been made in various sites. We begin by assuming that the flow is locally the same as it would be in an infinite circular tube of simple elastic properties; we split the problem up into small parts, and indicate how they can be put together.

## 2. The propagation of waves in a fluid-filled elastic tube.

Consider an infinite, uniform elastic tube in which an incompressible inviscid fluid flows uniformly with velocity $U_0$ (the lack of viscosity means that the fluid provides only inertia, not damping). Consider small deformations of the wall, slowly varying with x, the distance along it; this is the same as the long wavelength approxiamtion: radius $\ll$ wavelength.

Let the velocity, pressure and cross-sectional area be $u(x,t) = U_o + u'(x,t)$, $p(x,t)$ and $A(x,t)$ respectively. Then the governing equations are:

x - momentum: $$u_t + u\, u_x = -\frac{1}{\rho}\, p_x \qquad (1)$$

continuity: $$A_t + (uA)_x = 0 \qquad (2)$$

elasticity: $$A \equiv A(p) \quad \text{(say)} \qquad (3)$$

where $\rho$ is the fluid density.

Let us assume that the solution is a wave of wave-speed C, and that $u' \ll C$. Then the $\partial/\partial t$ term is of the sameorder as $C\,\partial/\partial x$ which is large compared with $u'\,\partial/\partial x$. The governing equations then become:

from (1): $$u'_t + U_o u'_x = -\frac{1}{\rho}\, p_x \qquad (4)$$

from (2) & (3): $$p_t + U_o p_x = -\left(\frac{A}{\partial A/\partial p}\right)_o u'_x \qquad (5)$$

where the suffix zero means that the quantity in the bracket is evaluated in the undisturbed state. Combine (4) and (5) introducing the new variable $\xi = x - U_o t$ in place of x, and obtain:

$$u_{tt} = C_o^2\, u_{\xi\xi} \qquad (6)$$

where $$C_o^2 = \frac{1}{\rho}\left(\frac{A}{\partial A/\partial p}\right)_o . \qquad (7)$$

This is the wave equation, with general solutions

$$u = f_1(\xi - C_o t) + f_2(\xi + C_o t)$$