Ground (reference)

$$p_c = \frac{mv_j}{r} \qquad \text{where } r = \text{const} = \frac{h-\frac{t}{2}}{1+\cos\theta} \tag{1}$$

Water (fig. 2)

$$AB : p_c - p_0 = \frac{mv_j}{r_{ec}} \qquad \text{where } r_{ec} = \text{const} = \frac{h_e-\frac{t}{2}}{1+\cos\theta} \tag{2}$$

$$BD : \left[p_c - \gamma\left(z+\frac{t}{2}\right)\right] - p_0 = \frac{mv_j}{r_e} \qquad \text{where } r_e = r_e(z) \tag{3}$$

$$r_e = \infty \quad \text{for } z = \frac{p_c-p_0}{\gamma} - \frac{t}{2} \equiv z_i \tag{4}$$

with (1) $\rightarrow$

$$r_e = \frac{p_c}{\gamma}\cdot\frac{h-\frac{t}{2}}{1+\cos\theta}\cdot\frac{1}{z_i-z} \tag{5}$$

$$(4)\ (2) \text{ and } (1) \rightarrow z_i = \frac{p_c}{\gamma}\cdot\frac{h-\frac{t}{2}}{h_e-\frac{t}{2}} - \frac{t}{2} \tag{6}$$

$$DE : p_0 = \frac{mv_j}{r_0} \qquad \text{where } r = \text{const} = \frac{h-\frac{t}{2}}{1-\cos(\bar{\phi}-\theta)} \tag{7}$$

fig. 3 $\rightarrow dz = r_e d\phi \sin\phi$ (5) $\rightarrow (z_i - z)dz = \dfrac{p_c(h-\frac{t}{2})}{\gamma(1+\cos\theta)}\cdot \sin\phi \; d\phi$ $\Longrightarrow$

integration with the initial condition (fig. 2): $\phi(z=0)=0$

$$\Longrightarrow \quad z\left(z_i - \frac{z}{2}\right) = \frac{p_c\left(h - \frac{t}{2}\right)}{\gamma(1+\cos\theta)}(1-\cos\phi) \quad \rightarrow \quad \cos\phi = 1 - 2k\sin^2\Phi \tag{8}$$

we call: $$\frac{\gamma z_i^2}{p_c\left(h - \frac{t}{2}\right)}(1+\cos\theta) \equiv 4k^2 \tag{8a}$$

$$\frac{2z_i z - z^2}{z_i^2} \equiv \sin^2\phi \tag{8b}$$

fig. 3 $\rightarrow$ $dx = ctg\phi \cdot dz$

(8b) $\rightarrow$ $\cos\Phi = 1 - \frac{z}{z_i}$ $\rightarrow$ $dz = z_i \sin\Phi d\Phi$

$$\rightarrow dx = \frac{z_i}{k} \frac{1 - k^2\sin^2\Phi - \frac{1}{2}}{1 - k^2\sin^2\Phi} d\Phi$$

point D (fig. 2), we call:

$$\Phi\left(z = \frac{p_c}{\gamma} - \frac{t}{2}\right) \equiv \overline{\Phi} \quad \rightarrow \quad \cos\overline{\Phi} = 1 - \frac{1}{z_i}\left(\frac{p_c}{\gamma} - \frac{t}{2}\right) \tag{9}$$

integration from $\Phi = 0$ to $\Phi = \overline{\Phi}$ $\Longrightarrow$

$$\Longrightarrow \overline{x} \equiv x\left(z = \frac{p_c}{\gamma} - \frac{t}{2}\right) \equiv x(\text{point D}):$$

$$= \frac{z_i}{k}\left(E(\overline{\Phi},k) - \frac{1}{2}F(\overline{\Phi},k)\right) \tag{10}$$

where E - elliptic integral of first kind

F - elliptic integral of second kind

We call: $\phi(\text{point D}) = \phi(z = \frac{p_c}{\gamma} - \frac{t}{2}) \equiv \bar{\phi}$ (8) $\Big| \rightarrow \cos\bar{\phi} = 1 - 2k^2 \sin^2\bar{\phi}$ (11)

fig. 2 $\rightarrow h_0 + e + \frac{t}{2} = (r_{ec}\sin\theta + \bar{x} - \Delta \operatorname{ctg}\theta)\cdot\sin\theta$ (12)

where (2) $\rightarrow r_{ec}$ and $\Delta = \frac{p_c}{\gamma} - h_e$

(1) and (2) $p_0 = mv_j(1+\cos\theta)\left(\frac{1}{h-\frac{t}{2}} - \frac{1}{h_e - \frac{t}{2}}\right)$ (7) $\Big| \rightarrow (h_0 - \frac{t}{2})\cdot\left(\frac{1}{h-\frac{t}{2}} - \frac{1}{h_e - \frac{t}{2}}\right) = \frac{1-\cos(\bar{\phi}-\theta)}{1+\cos\theta}$ (13)

Scheme of calculations:

(6) $\rightarrow z_i$ — (9) $\rightarrow \bar{\Phi}$; $\rightarrow$ (8a) $\rightarrow k$ (10) $\Big|$ $\rightarrow \bar{x}$; (2) $\rightarrow r_{ec}$ (12) $\Big|$ $\rightarrow h_0$; (11) $\rightarrow \bar{\phi}$; equation (13) $\Big|$ $\rightarrow h_e$

Coordinate $x = x(z)$ of the jet mean line is given by:

$$x = \frac{z_i}{k}\left(\int_0^{\Phi(z)} \sqrt{1-k^2\sin^2\Phi}\, d\Phi - \frac{1}{2}\int_0^{\Phi(z)} \frac{d\Phi}{\sqrt{1-k^2\sin^2\Phi}}\right)$$

where $\Phi = \arccos\,(1 - \frac{z}{z_i})$