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INTRODUCTION TO COMPUTATIONAL FLUID DYNAMICS

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INTRODUCTION TO FINITE ELEMENT TECHNIQUES IN FLUID MECHANICS

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1. STRONG AND WEAK FORMULATIONS OF A BOUNDARY VALUE PROBLEM.

1.1. Strong formulation.

Consider as an example, the following simple one-dimensional boundary value problem, consisting of the *differential equation* :

$$\frac{d}{dx}(\lambda \frac{du}{dx}) = f \quad \text{on} \quad 0 \leq x \leq X \tag{1}$$

and the *boundary conditions*

$$u(0) = u_0 \tag{2}$$

and

$$\lambda \frac{du}{dx}(X) = q \tag{3}$$

More generally, in het sequel, the differential equation is denoted by

$$a(u) = f \tag{4}$$

the *domain* to which it applies is genoted by Ω. The boundary condition of type (2) usually is called a *Diriclet boundary condition*. In the sequel, more generally it is denoted by :

$$b_0(u) = g_0 \tag{5}$$

The boundary condition of type (3), which is formulated on the *flux* of the variable, is called a *Neumann boundary condition*. In the sequel, more generally it is denoted by :

$$b_1(u) = g_1 \tag{6}$$

The boundary of the domain Ω is denoted by $\partial\Omega$. The part to which the Diriclet boundary condition applied is denoted by $\partial\Omega_0$. The part to which the Neumann boundary condition applies is denoted by $\partial\Omega_1$.

The *boundary value problem* (1-3) is said to be in its *strong form*, requiring the satisfaction of the differential equation (1) in all points of the domain Ω, the satisfaction of the Diriclet boundary condition (2) in all points (here one) of the part of the boundary $\partial\Omega_0$ and the satisfaction of the Neumann boundary condition (3) in all points (here one) of the part of the boundary $\partial\Omega_1$.

One way of relaxing the requirements of the boundary value problem, namely the *finite difference way*, consists in requiring the approximate satisfaction of the differential equation and the boundary conditions in a finite number of points in the domain and at the boundary. These points usually are chosen to belong to a *mesh* with some form of regularity. For the one-dimensional domain, a typical mesh or grid is obtained by choosing equally spaced *grid points*, as shown on figure 1.